There is More Than One Way to *DO* Math?!

Formative Assessment and Conceptual Understanding with the Enhanced Learning Maps Project

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Enhanced Learning Maps Project Goal

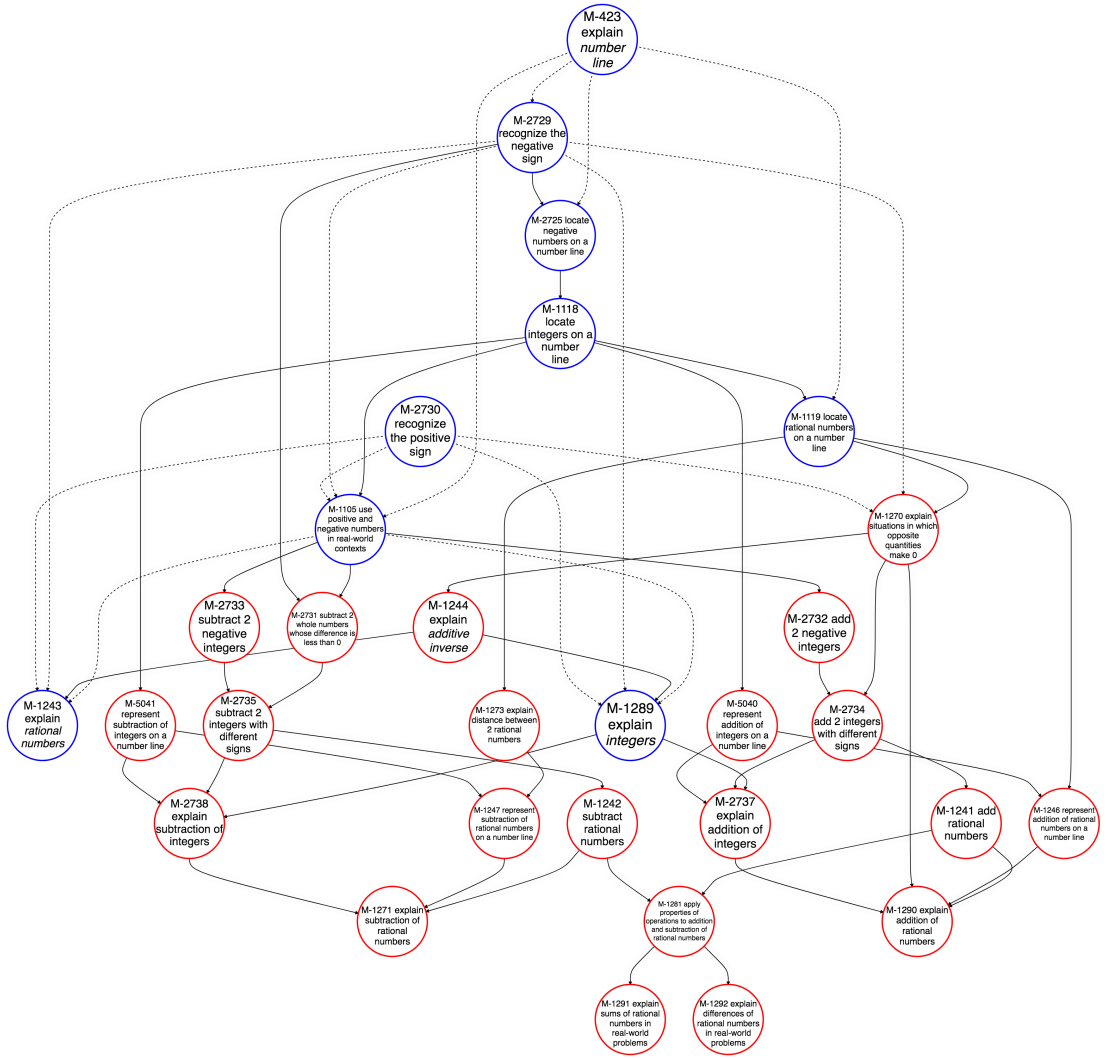
To improve teachers' ability to use effective formative assessment tools and practices to provide personalized instruction resulting in greater student achievement.

Many paths to student understanding…

- Different start and end points
- Different routes
- Different gaps along the way

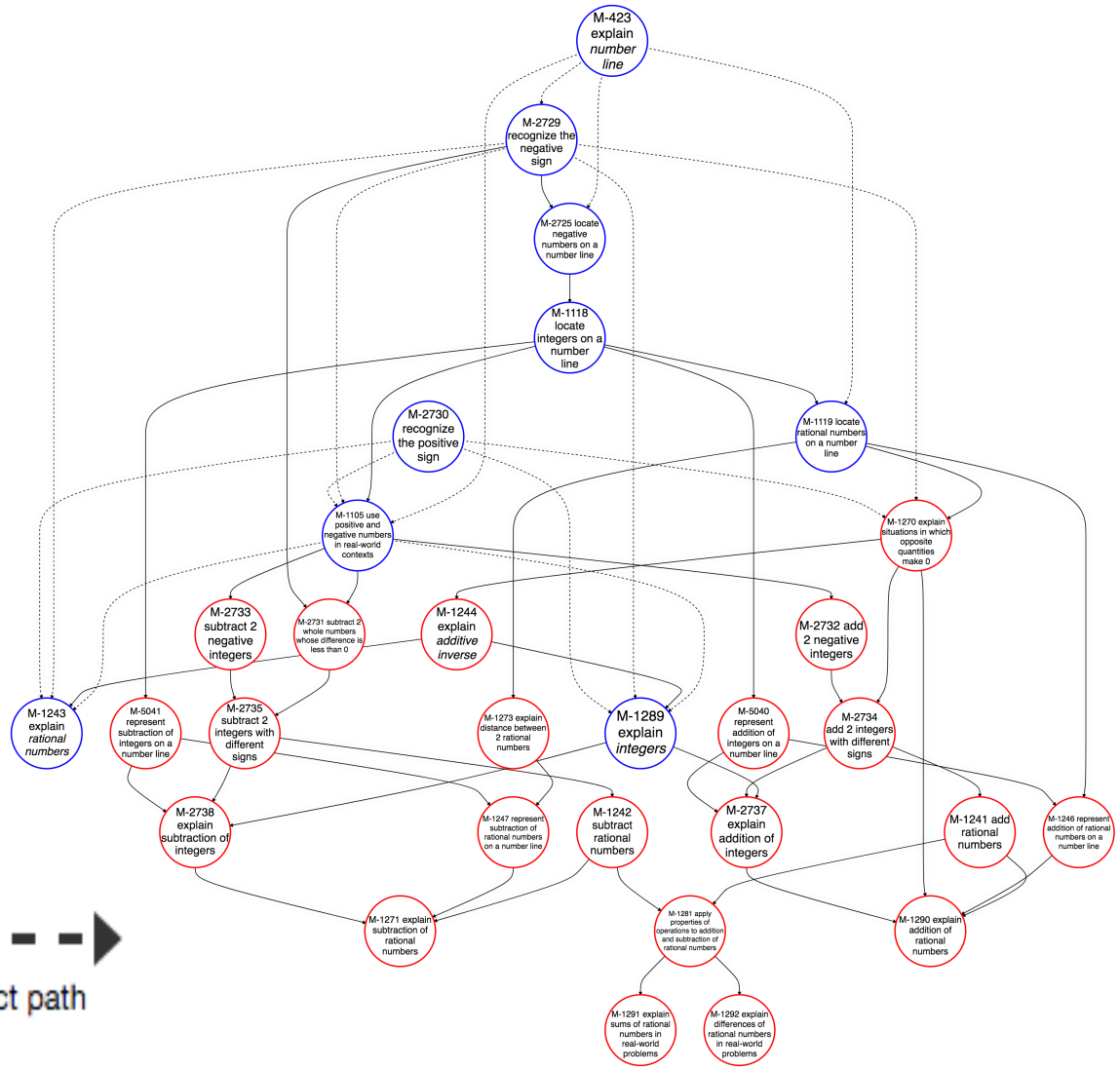
Enhanced Learning Map Model

- **Visual display** of concepts and skills
- **Connections** show prerequisites and successor skills
- **Multiple pathways**
- **Granular detail**



Map Views

- Nodes
- Connections
- Standards
- Research based
- Teacher selected



Target skill

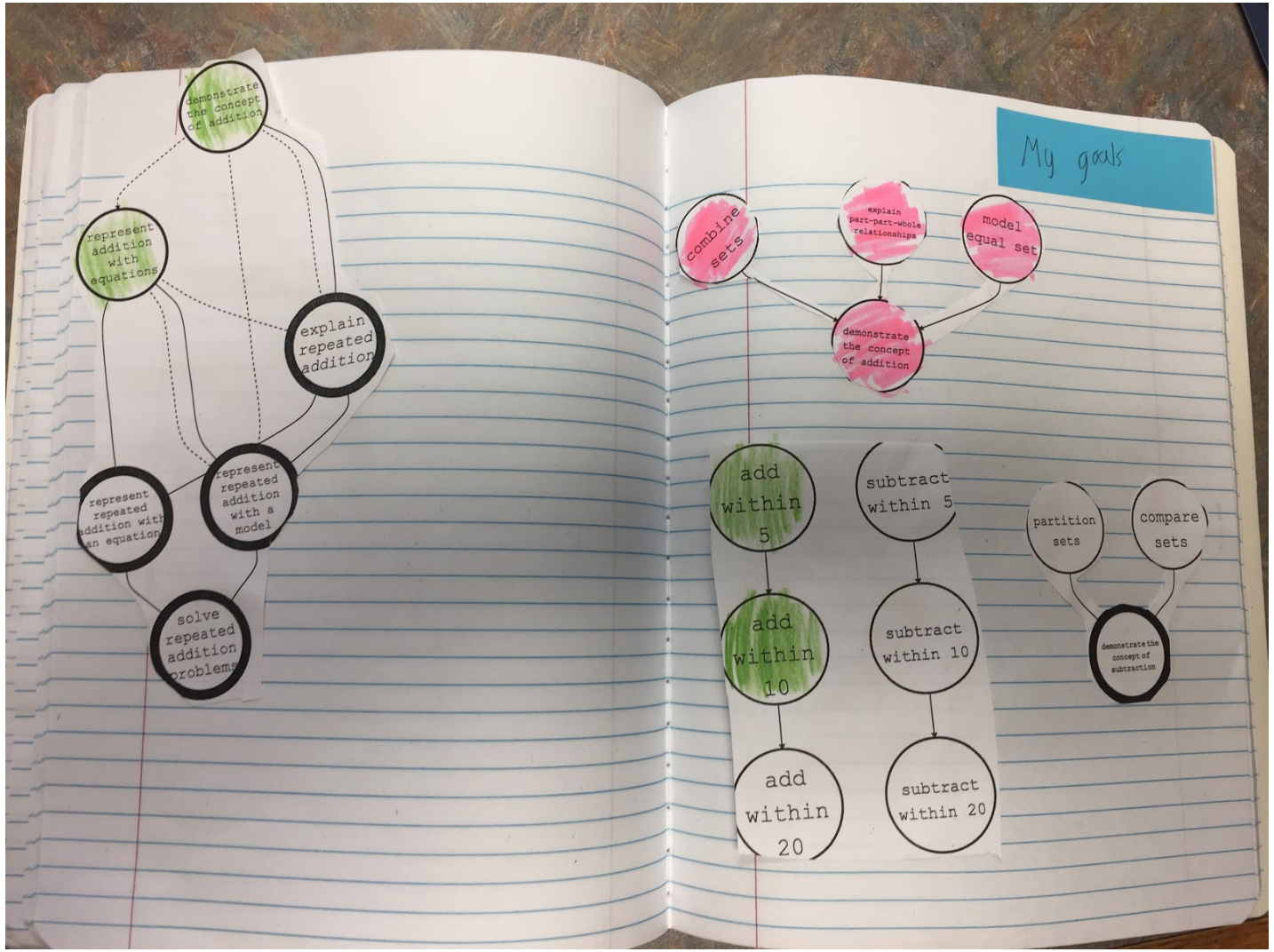
Prerequisite or related skill

Direct path

Indirect path

Multiple Pathways of Learning

Teachers Using the Maps



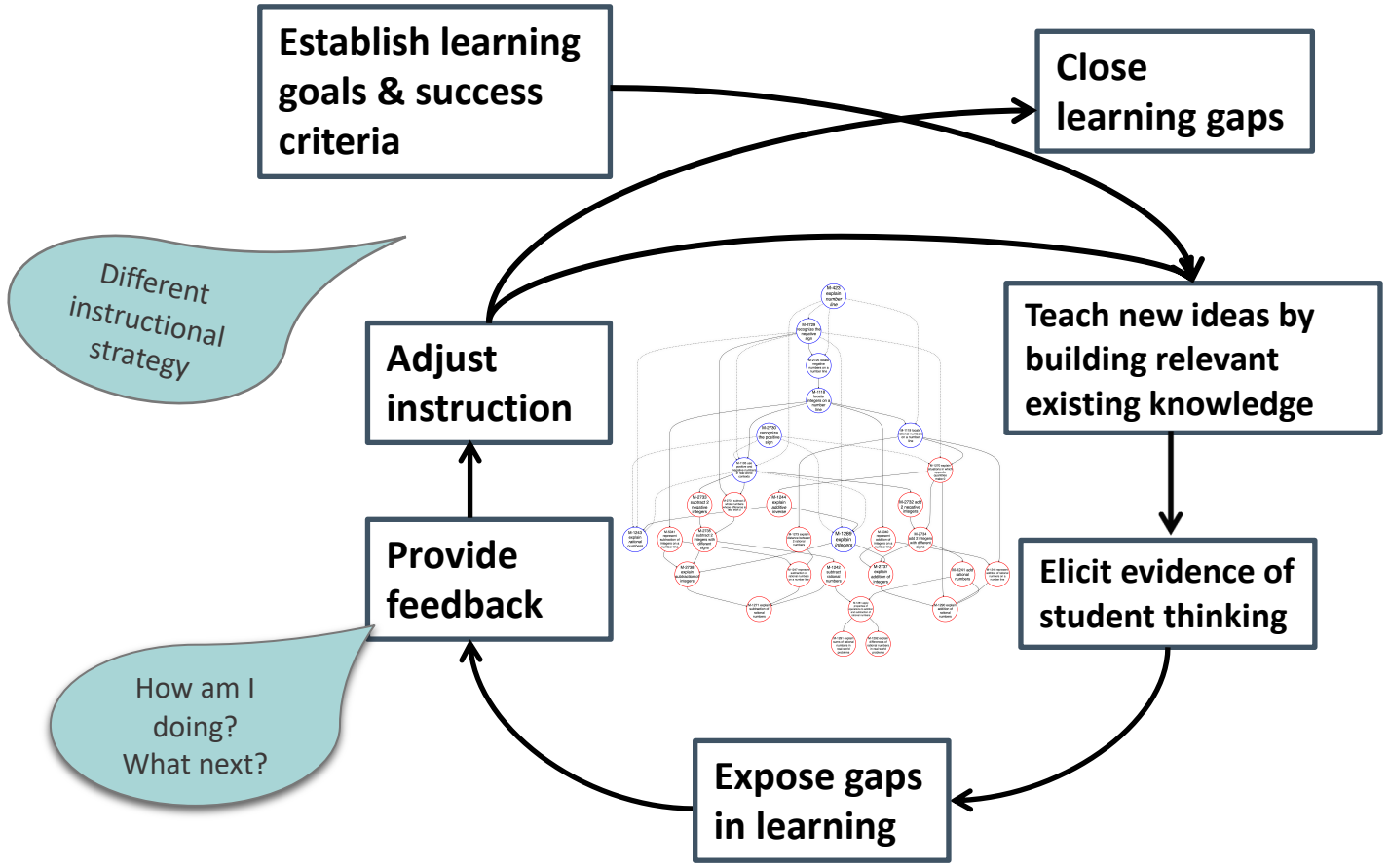
The map alone will not move students forward

Learning Map Model
Visual representation of how students learn

Formative Assessment
Approach to teaching that is a process for moving students forward

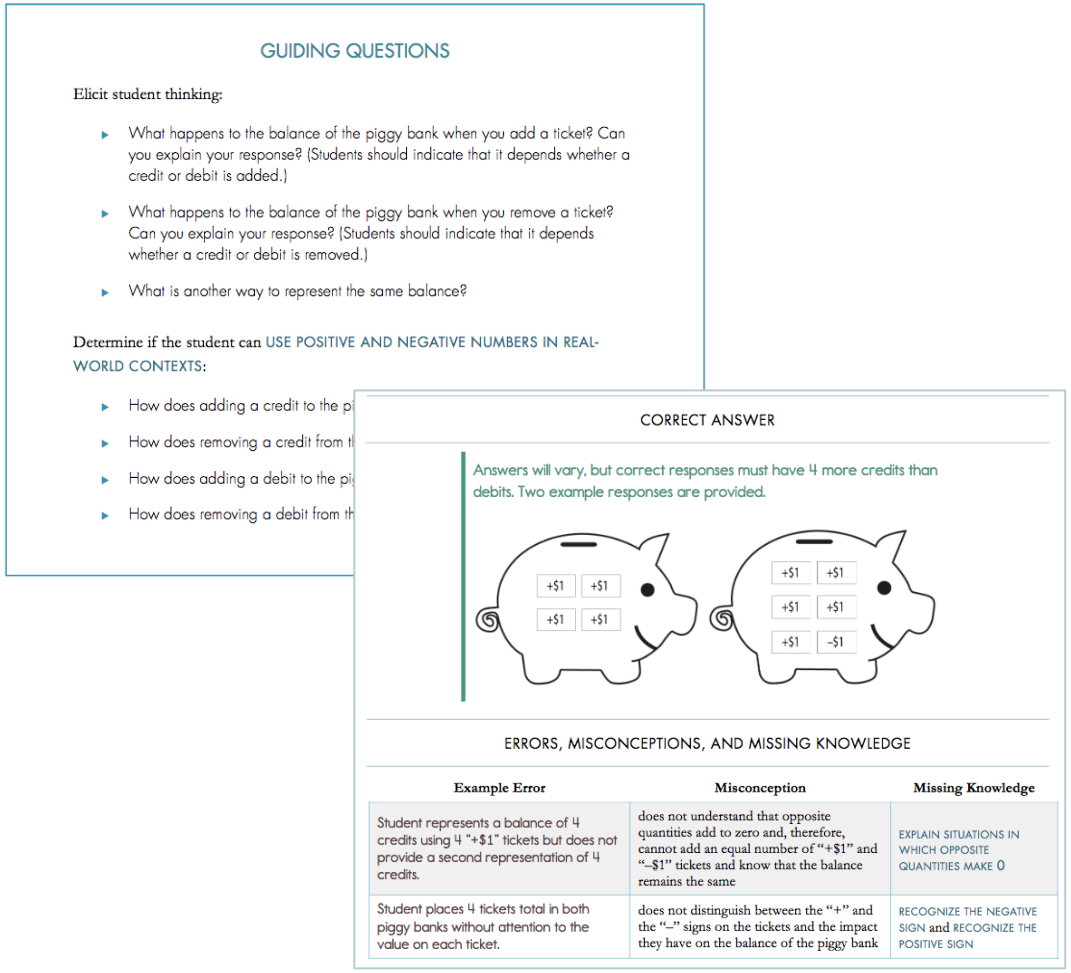
Formative Assessment

- A process, NOT a test
- Aims to reveal student thinking
- Occurs within instruction, moment-to-moment
- Continuous, iterative
- Informs instructional decisions



Enhanced Learning Map Models and Formative Assessment

- **Clarify** learning goals
- **Promote** effective learning by focusing on connections
- **Determine** where students are in their learning and **move** them forward



GUIDING QUESTIONS

Elicit student thinking:

- What happens to the balance of the piggy bank when you add a ticket? Can you explain your response? (Students should indicate that it depends whether a credit or debit is added.)
- What happens to the balance of the piggy bank when you remove a ticket? Can you explain your response? (Students should indicate that it depends whether a credit or debit is removed.)
- What is another way to represent the same balance?

Determine if the student can USE POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS:

- How does adding a credit to the pi
- How does removing a credit from t
- How does adding a debit to the pi
- How does removing a debit from th

CORRECT ANSWER

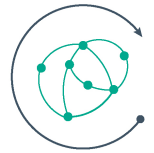
Answers will vary, but correct responses must have 4 more credits than debits. Two example responses are provided.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student represents a balance of 4 credits using 4 "+$1" tickets but does not provide a second representation of 4 credits.	does not understand that opposite quantities add to zero and, therefore, cannot add an equal number of "+$1" and "–$1" tickets and know that the balance remains the same	EXPLAIN SITUATIONS IN WHICH OPPOSITE QUANTITIES MAKE 0
Student places 4 tickets total in both piggy banks without attention to the value on each ticket.	does not distinguish between the "+" and the "–" signs on the tickets and the impact they have on the balance of the piggy bank	RECOGNIZE THE NEGATIVE SIGN and RECOGNIZE THE POSITIVE SIGN

Formative Assessment

Informed Instructional Framework



Formative Assessment
Approach to teaching that is a process for moving students forward

+

Learning Map Model
Visual representation of how students learn

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Instructional Resources
Set learning goals and create the conditions for noticing where students are

Instructional Resources include:

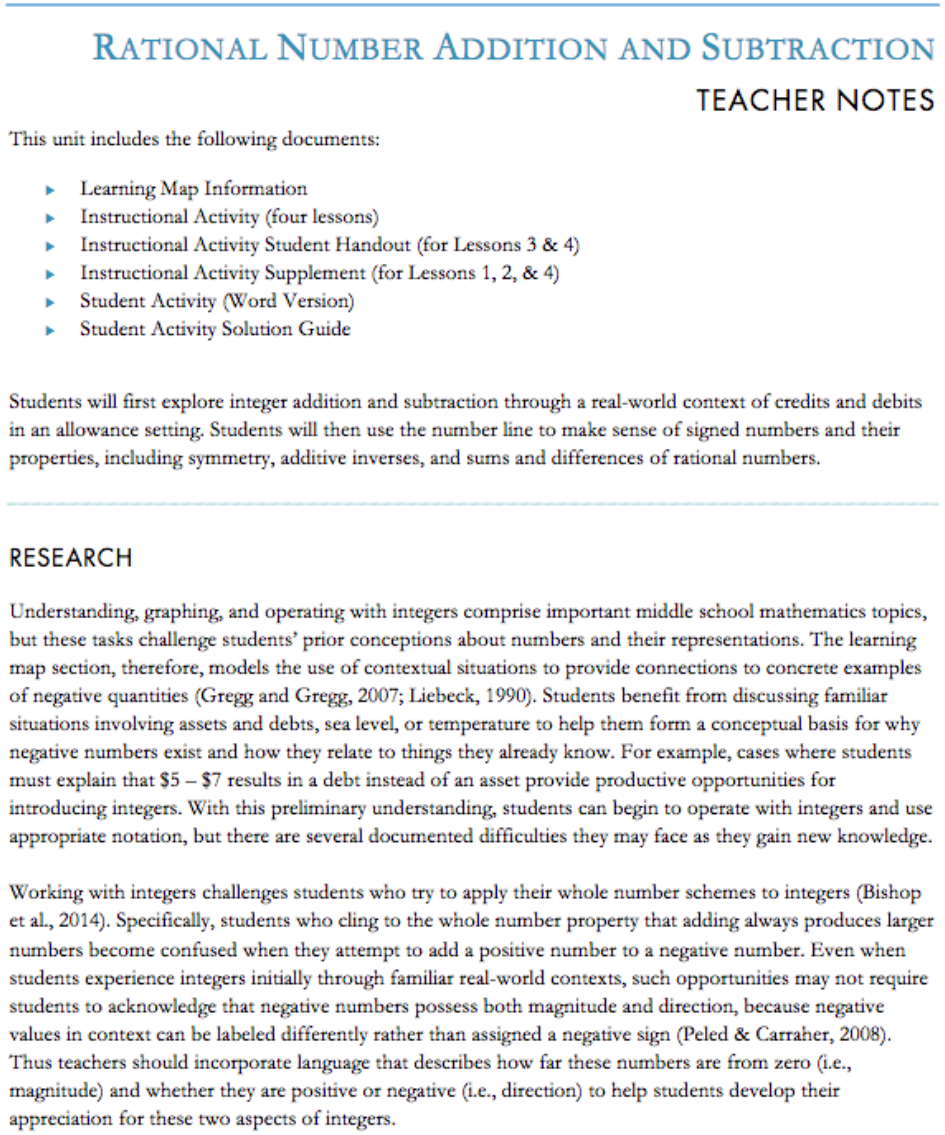
- Learning Map Model document
- Teacher Notes
- Instructional Activities
- Student Activity
- Solution/Feedback Guide

Learning Map Models and Mathematics Teaching Practices

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

Teacher Notes

- Summary of current, peer-reviewed, relevant research
- Include:
  - best practices
  - instructional strategies
  - common misconceptions & errors – and how to avoid or deter them
  - reference list
- Teacher Notes Videos offer additional support
  - Approximately 3 minutes
  - One for each grade level
  - Can be used in classroom or just as a teacher resource



Rational Number Addition and Subtraction

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (four lessons)
- Instructional Activity Student Handout (for Lessons 3 & 4)
- Instructional Activity Supplement (for Lessons 1, 2, & 4)
- Student Activity (Word Version)
- Student Activity Solution Guide

Students will first explore integer addition and subtraction through a real-world context of credits and debits in an allowance setting. Students will then use the number line to make sense of signed numbers and their properties, including symmetry, additive inverses, and sums and differences of rational numbers.

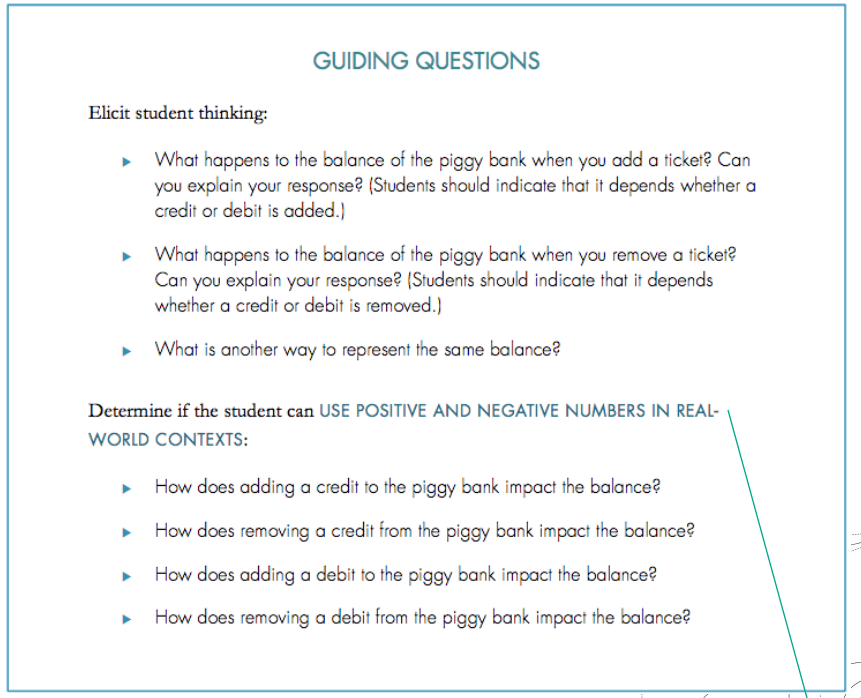
RESEARCH

Understanding, graphing, and operating with integers comprise important middle school mathematics topics, but these tasks challenge students' prior conceptions about numbers and their representations. The learning map section, therefore, models the use of contextual situations to provide connections to concrete examples of negative quantities (Gregg and Gregg, 2007; Liebeck, 1990). Students benefit from discussing familiar situations involving assets and debts, sea level, or temperature to help them form a conceptual basis for why negative numbers exist and how they relate to things they already know. For example, cases where students must explain that $5 – $7 results in a debt instead of an asset provide productive opportunities for introducing integers. With this preliminary understanding, students can begin to operate with integers and use appropriate notation, but there are several documented difficulties they may face as they gain new knowledge.

Working with integers challenges students who try to apply their whole number schemes to integers (Bishop et al., 2014). Specifically, students who cling to the whole number property that adding always produces larger numbers become confused when they attempt to add a positive number to a negative number. Even when students experience integers initially through familiar real-world contexts, such opportunities may not require students to acknowledge that negative numbers possess both magnitude and direction, because negative values in context can be labeled differently rather than assigned a negative sign (Peled & Carraher, 2008). Thus teachers should incorporate language that describes how far these numbers are from zero (i.e., magnitude) and whether they are positive or negative (i.e., direction) to help students develop their appreciation for these two aspects of integers.

Instructional Activities
Guiding Questions

- Unique component of the ELM resources
- Link back to the learning map model
- Bring formative assessment into each lesson
- Allow teacher to adjust instruction based on student response



GUIDING QUESTIONS

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Determine if the student can USE POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS:

- How does adding a credit to the piggy bank impact the balance?
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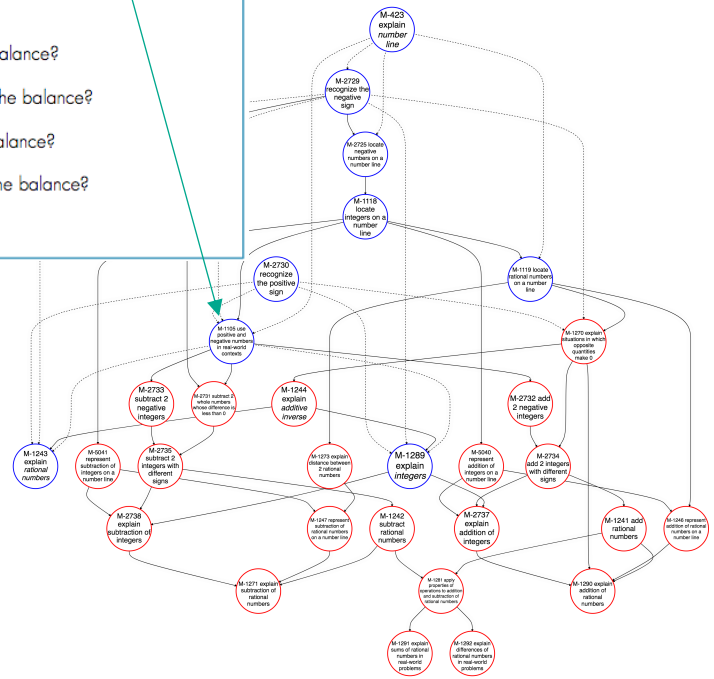
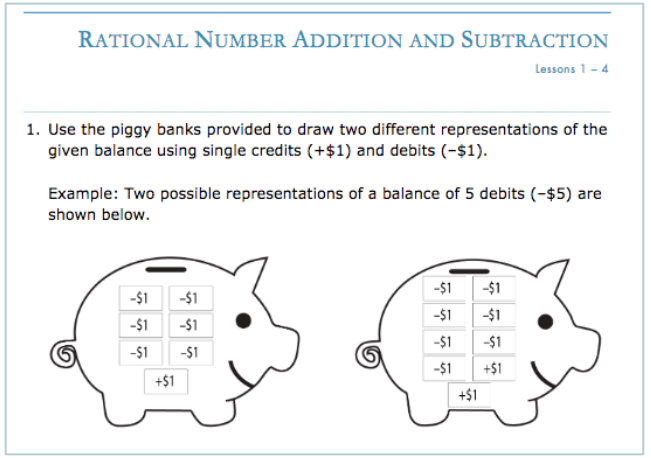


Guiding Questions

Instructional Activities
Student Activity & Solution Guide

- Independent task
- Solution Guide link back to the learning map model
- Bring formative assessment into each unit or lesson
- Allow teacher to adjust instruction based on student response
- Flexible implementation



RATIONAL NUMBER ADDITION AND SUBTRACTION

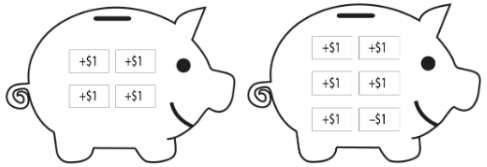
Lessons 1 – 4

1. Use the piggy banks provided to draw two different representations of the given balance using single credits (+$1) and debits (−$1).

   Example: Two possible representations of a balance of 5 debits (−$5) are shown below.

CORRECT ANSWER

Answers will vary, but correct responses must have 4 more credits than debits. Two example responses are provided.



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
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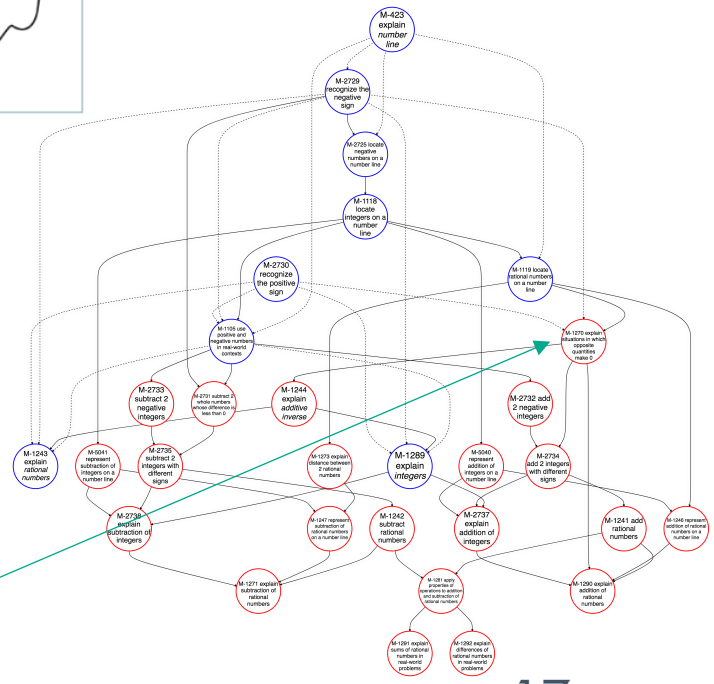
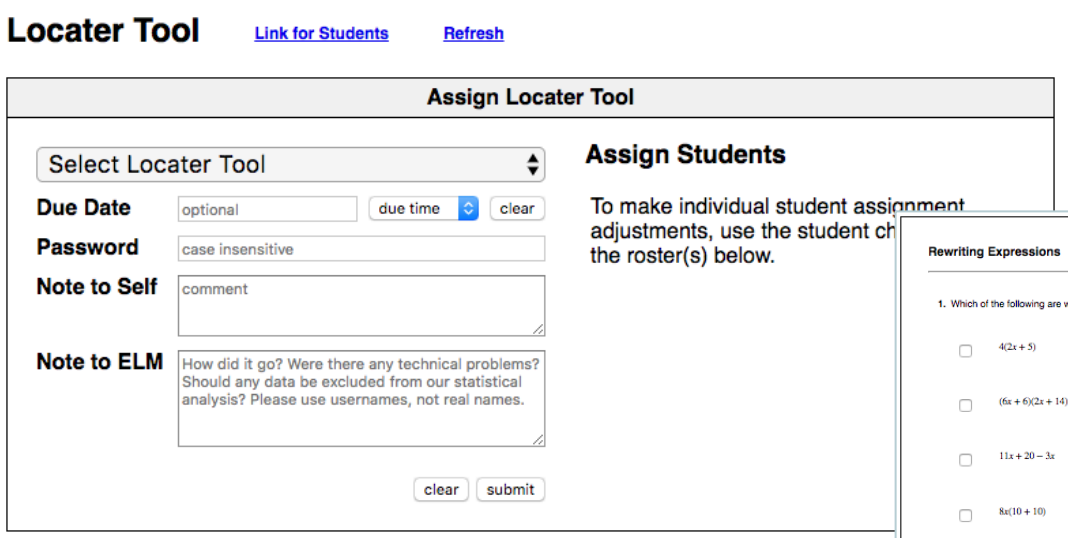
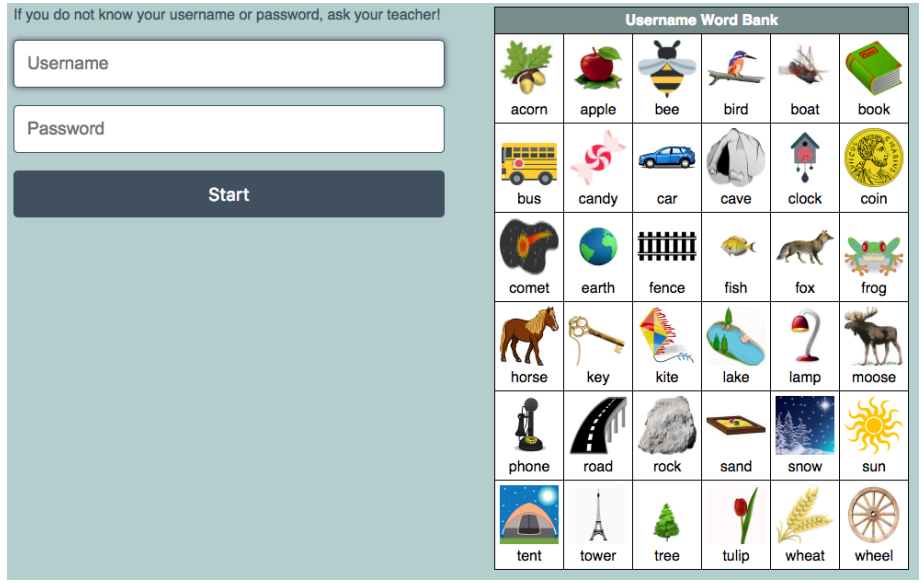
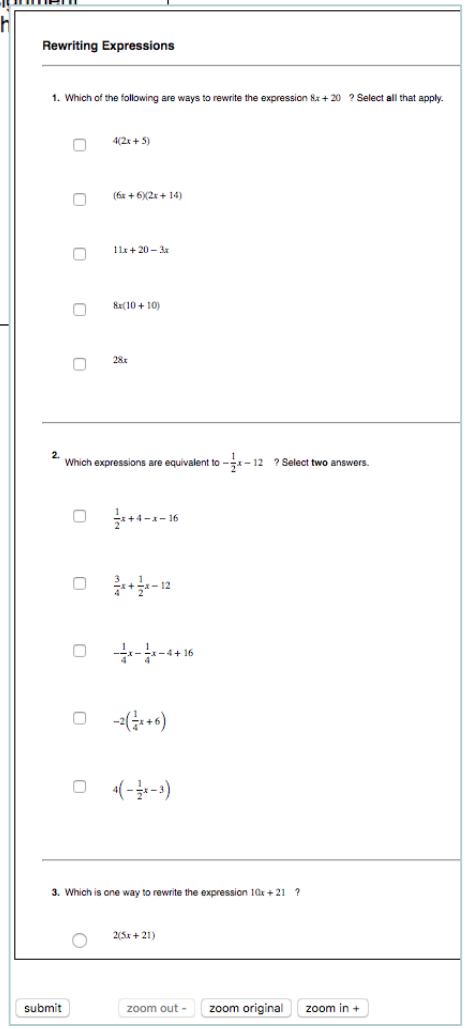


Locater Tool

- a device or system used for determining the position or location of something
- Grounded in the map model and guided by the instructional units
- Assist teachers in creating personalized learning progressions/maps for students





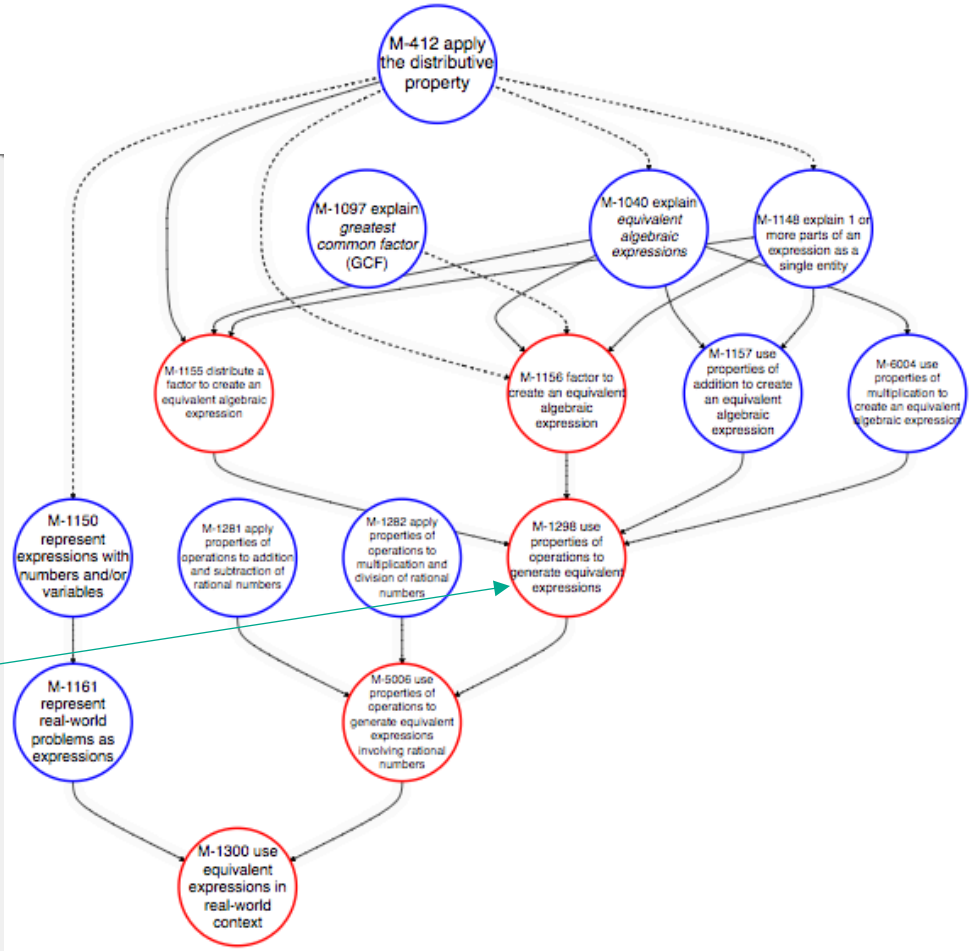


Locater Tool

Item Report Viewer print export to CSV x

2. Which expressions are equivalent to `-1/2 x - 12`? Select **two** answers.

Option	Understood node(s)	Misunderstood node(s)	% answered
A. `1/2 x + 4 - x - 16` ✓	use properties of operations to generate equivalent expressions involving rational numbers		100%
B. `3/4 x + 1/2 x - 12`		apply properties of operations to addition and subtraction of rational numbers	0%
C. `-1/4 x - 1/4 x - 4 + 16`		use properties of operations to generate equivalent expressions	100%
D. `-2(1/4 x + 6)` ✓	use properties of operations to generate equivalent expressions involving rational numbers		0%
E. `4(-1/2 x - 3)`		factor to create an equivalent algebraic expression	0%



Teacher video clip

ENHANCED LEARNING MAPS
INSIGHTS FOR INSTRUCTION

Enhanced Learning Maps Software

elmap.us

User name: jayhawk1@ku.edu

Password: mctm

Questions?

THE CENTER FOR ASSESSMENT AND ACCOUNTABILITY RESEARCH AND DESIGN
The University of Kansas

Interested in learning more or participating in the project?

- Contact Enhanced Learning Maps project staff

EnhancedLM@ku.edu

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