MUTIPLY MULTI-DIGIT WHOLE NUMBERS

5.NBT.5

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

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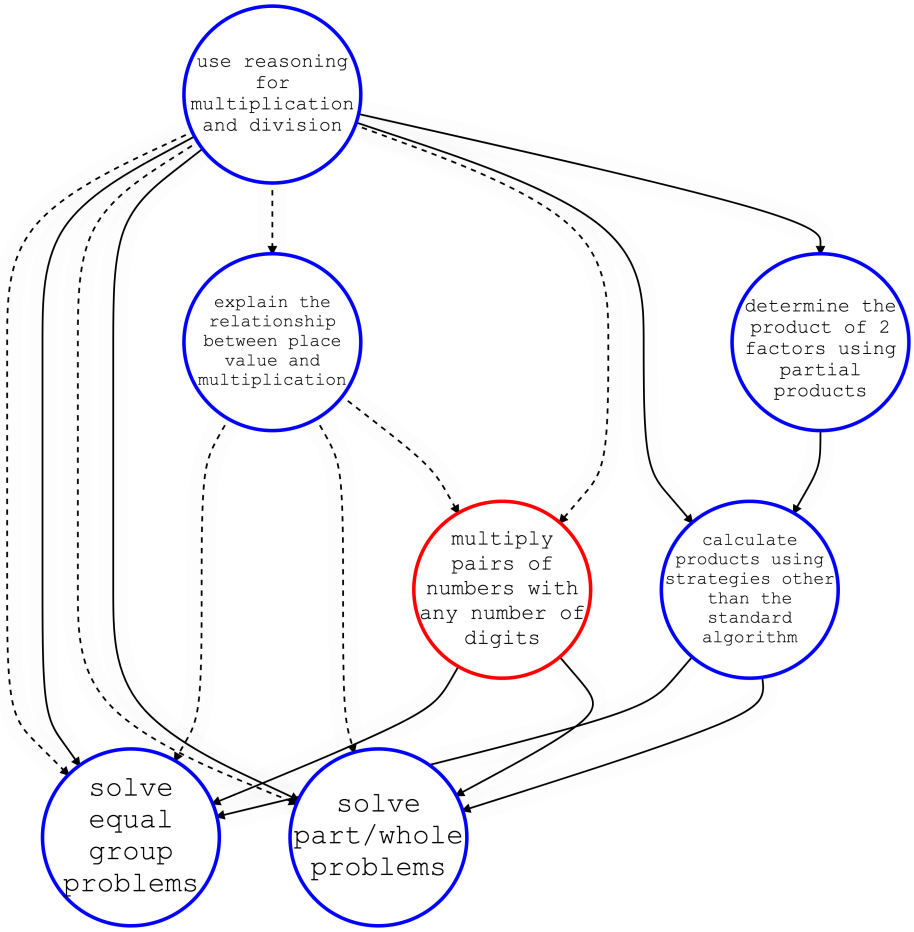
MULTIPLY MULTI-DIGIT WHOLE NUMBERS

LEARNING MAP INFORMATION

STANDARDS

5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.



**Learning map model of* 5.NBT.5

Node Name	Node Description
CALCULATE PRODUCTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM	Demonstrate multiplication of two factors using words, drawings, manipulatives, or other strategies such as partial products.
DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS	Determine the product of two factors by partitioning the factors into partial products according to base-ten properties.
EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION	Make known your understanding that multi-digit numbers represent quantities of ones, tens, hundreds, etc., and that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right, and use this understanding to explain how to calculate the products of multi-digit factors.
MULTIPLY PAIRS OF NUMBERS WITH ANY NUMBER OF DIGITS	Determine the product of pairs of numbers with any number of digits.
USE REASONING FOR MULTIPLICATION AND DIVISION	Solve multiplication and division problems strategically.
SOLVE EQUAL GROUP PROBLEMS	Use multiplication and division to solve word problems in situations involving equal sets.
SOLVE PART/WHOLE PROBLEMS	Use multiplication and division to solve word problems in situations involving parts and wholes.

ADDITIONAL NODES RELATED TO THIS UNIT OF INSTRUCTION

Node Name	Node Description	Related Node
APPLY THE DISTRIBUTIVE PROPERTY	Apply distributive property as a strategy for multiplication. For example, 7 × 18 can be calculated as (7 × 10) + (7 × 8).	Prerequisite of USE REASONING FOR MULTIPLICATION AND DIVISION (through EXPLAIN THE DISTRIBUTIVE PROPERTY)
DECOMPOSE NUMBERS UP TO 1,000	Break apart a larger number up to 1,000 to form a smaller number.	Prerequisite of EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION (through EXPLAIN PLACE VALUE FOR ONES AND TENS and DECOMPOSE NUMBERS BASED ON TENS)
EXPLAIN THE DISTRIBUTIVE PROPERTY	Make known your understanding that when multiplying a number over a sum, you distribute the factor to each addend (i.e., multiply the factor by each addend and then add).	Prerequisite of USE REASONING FOR MULTIPLICATION AND DIVISION
MULTIPLY A 2-DIGIT NUMBER BY A 1-DIGIT NUMBER WITHOUT REGROUPING	Multiply a two-digit number by a one-digit number using regrouping. The two-digit number cannot be a multiple of ten (e.g., 24 × 3).	Prerequisite of MULTIPLY PAIRS OF NUMBERS WITH ANY NUMBER OF DIGITS (through MULTIPLY A 3-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING and MULTIPLY A 2-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING and MULTIPLY A 2-DIGIT NUMBER BY A 1-DIGIT NUMBER WITH REGROUPING)

Node Name	Node Description	Related Node
MULTIPLY A 2-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING	Multiply a two-digit number by a two-digit number using regrouping (e.g., 57 × 28).	Prerequisite of MULTIPLY PAIRS OF NUMBERS WITH ANY NUMBER OF DIGITS (through MULTIPLY A 3-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING)
MULTIPLY A 2-DIGIT NUMBER BY A 2-DIGIT NUMBER WITHOUT REGROUPING	Multiply a two-digit number by a two-digit number without using regrouping (e.g., 32 × 32).	Prerequisite of MULTIPLY PAIRS OF NUMBERS WITH ANY NUMBER OF DIGITS (through MULTIPLY A 3-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING and MULTIPLY A 2-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING)
MULTIPLY A 3-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING	Multiply a three-digit number by a two-digit number using regrouping (e.g., 495 × 56).	Prerequisite of MULTIPLY PAIRS OF NUMBERS WITH ANY NUMBER OF DIGITS
MULTIPLY 1-DIGIT WHOLE NUMBERS BY MULTIPLES OF 10	Multiply one-digit whole numbers by multiples of ten (e.g., 7 × 20).	Prerequisite of MULTIPLY PAIRS OF NUMBERS WITH ANY NUMBER OF DIGITS (through MULTIPLY A 3-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING and MULTIPLY A 2-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING and MULTIPLY A 2-DIGIT NUMBER BY A 1-DIGIT NUMBER WITH REGROUPING and MULTIPLY A 2-DIGIT NUMBER BY A 1-DIGIT NUMBER WITHOUT REGROUPING)
SOLVE CARTESIAN PRODUCTS	Use multiplication to solve word problems in situations involving Cartesian products. A Cartesian product is a multiplication problem that shows the combination of two sets A and B, where one element from set A is matched with an element from set B. For example, if Jenny has seven shirts and four skirts, how many different outfits can she make?	Postrequisite of MULTIPLY PAIRS OF NUMBERS WITH ANY NUMBER OF DIGITS
SOLVE MULTIPLICATIVE COMPARISON PROBLEMS	Use multiplication and division to solve word problems involving multiplicative comparison.	Postrequisite of MULTIPLY PAIRS OF NUMBERS WITH ANY NUMBER OF DIGITS
SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES	Solve real-world and contextualized mathematical problems involving area of rectangles.	Postrequisite of EXPLAIN THE DISTRIBUTIVE PROPERTY (through CALCULATE AREA OF IRREGULAR SHAPES BY PARTITIONING INTO RECTANGLES)

MULTIPLY MULTI-DIGIT WHOLE NUMBERS

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (3 lessons)
- Instructional Activity Student Handout (for Lessons 1-3)
- Instructional Activity Supplement (for Lessons 1-3)
- Student Activity (Word Version)
- Student Activity Solution Guide

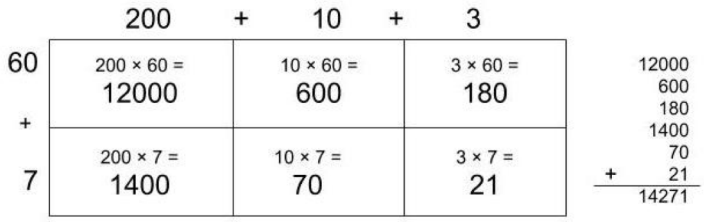
In these lessons, students will learn how to multiply multi-digit whole numbers fluently using a variety of strategies including the standard algorithm. By using a variety of strategies to solve multi-digit multiplication problems, students increase their understanding of multiplication and improve their problem solving capabilities (Fuson, 2003). The activities in these lessons attend to students' developing computational fluency by flexibly using various computational methods and explaining those methods (NCTM, 2000, p. 152).

Fuson (2003) and other researchers noted that students should have opportunities to wrestle with problems and contexts, and to build solutions using available strategies. Often such strategies rely on their prior knowledge of addition. Baek (2006) identified several invented strategies students applied as they confronted situations that required multiplication. These strategies are doubling, single-factor partitioning, and compensating. Each of the invented strategies involve using partial products to calculate the total product. Students that use a doubling strategy pair up sets of one factor until they have as many pairs or sets as the second factor. The single-factor partitioning strategies involve decomposing or partitioning one of the factors in order to calculate the product. The compensating strategies require a student to round a factor, usually to a decade number, to multiply and then compensate later by adding to or subtracting from the total product. Area models require students to decompose values and calculate partial products before calculating a total product.

These lessons will focus on the use of area models, beginning with rectangular arrays. Students will decompose multi-digit numbers, multiply partial products, and then add the partial products to calculate a total product. For example, with the expression 67×213, 213 would be decomposed as $200 + 10 + 3$, and 67 would be decomposed $60 + 7$. The decomposed values would be written outside the area model; the partial products would then be written inside the area model. Once all partial products are calculated, they would be added outside the area model to determine the total product.

AN EXAMPLE

Simplifying 67 × 213 using an area model.



Students will draw on their understanding of arrays to create area models that show the relationships between the factors being multiplied and how they can be grouped in the multiplication process. When using an array or area model to multiply, students can visualize the quantities, creating a concrete foundation which will support student understanding of the concept of multiplication with multi-digit numbers (Fuson, 2003). Area models also provide students with a visual representation of the commutative and distributive properties of multiplication due to the row and column structure (Van de Walle, Karp, Lovin, & Bay-Williams, 2006).

The commutative property is shown in an area model through the fact that regardless of which factor is labeled as a rectangle's length or width, the area and shape of the resulting rectangle (product) will remain the same (Flowers, Krebs, & Rubenstein, 2006). This can be shown visually by constructing an area model with one factor listed along the vertical side and then turning the area model so that the same factor is now listed along the horizontal side; the area and shape remain constant before and after the rotation. This visual display reinforces the idea behind the commutative property of multiplication, that is, that the order of the factors does affect the product.

A second advantage of using area models is their clear visualization of the distributive property, which can reinforce student understanding of decomposing factors in different ways and multiplying to find several partial products as a strategy for calculating the total product. In particular, decomposing the factors based on place value will support student understanding of how and why the standard algorithm works (Fuson, 2003).

Fuson (2003) recommends that teachers incorporate problem solving opportunities early in their instruction, as it cultivates computational competence and builds problem solving skills. Some understanding of problem situations, along with experiences using different strategies to multiply, prompt students to reason through problems rather than to merely pick the numbers out and follow a learned routine. Useful multiplication contexts include equal group problems, Cartesian products, part/whole problems, and area problems. Students who work through problem situations while learning different multiplication methods—as opposed to the end of the lesson *after* the student has learned how to multiply fluently—experience more authentic applications of multiplication and develop an understanding of which situations require multiplication, which

results in students who are more confident in their critical thinking and problem solving abilities (Fuson, 2003).

The Learning Map section for this sequence of activities, 5.NBT.5, focuses on understanding place value and its relationship to multiplication by using a variety of strategies to calculate products, by reasoning about multiplication, and by solving a variety of problem types. Reasoning about multiplication requires students to understand a variety of strategies, as well as an understanding of when to apply each strategy. In order for students to reason about what types of strategies to use and when, they need to be able to calculate products using a variety of strategies. Throughout these lessons, students will be solving equal group problems, array problems, measurement quantities problems, multiplicative comparative problems, Cartesian products, and part/whole problems. First, in order to successfully calculate products using any method, students need to have a foundational understanding of place value and how it is related to multiplication. Emphasizing place value helps students consider a number's value—as opposed to viewing each digit in isolation—and is a key component of using base ten language (i.e., three ones times five tens is 15 tens). Students who can use this mathematical language to describe numbers and numerical relationships may develop fewer misconceptions, especially as they work toward the standard algorithm (Van de Walle et al., 2006).

The activities in these lessons are designed to build on student understanding of the place value system and its relationship with multiplication. Each lesson incorporates problem situations with the instruction of a multiplication strategy to help students create an understanding of the everyday situations which require multiplication. The lessons gradually build on a concrete understanding of multiplication (area model) to a more advanced representation of multiplication (standard algorithm). The idea is that a student can explain their understanding of the standard algorithm by showing their comprehension with an alternate strategy. The activities will also show students that some strategies are more beneficial than other strategies in certain situations and with certain numbers.

Teachers should be mindful to include examples that might draw out common misconceptions, especially in regard to place value and regrouping with the standard algorithm, in order to have the opportunity to explain, clarify, or correct the misconceptions. Each lesson incorporates an opportunity for the teacher to address errors and misconceptions with the whole class, small groups, or individual students. Guiding questions are also included to support teachers' questioning to determine student misconceptions.

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Van de Walle, J.A., Karp, K.S., Lovin, L.H., & Bay-Williams, J.M. (2006). *Teaching Student-Centered Mathematics*. Upper Saddle River, NJ: Pearson Professional Development.

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MULTIPLY MULTI-DIGIT WHOLE NUMBERS

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will multiply multi-digit whole numbers using partial products.	▶ DECOMPOSE NUMBERS UP TO 1,000 ▶ DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS ▶ CALCULATE PRODUCTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM ▶ EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION
Lesson 2	Students will fluently multiply multi-digit whole numbers using the distributive property and partial products.	▶ EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION ▶ MULTIPLY PAIRS OF NUMBERS WITH ANY NUMBER OF DIGITS
Lesson 3	Students will fluently evaluate multi-digit multiplication using the standard algorithm.	▶ EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION ▶ MULTIPLY A 2-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING ▶ MULTIPLY A 3-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING

MULTIPLY MULTI-DIGIT WHOLE NUMBERS

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will multiply multi-digit whole numbers using partial products.

PRIMARY ACTIVITY

Students will use an area model to decompose and multiply multi-digit whole numbers.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Factor
- Product
- Partial product
- Decompose

MATERIALS

- Base-ten blocks
- Dry erase board and markers or paper and pencil
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every two students.)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- Word Version INSTRUCTIONAL ACTIVITY SUPPLEMENT
- Word Version INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

IMPLEMENTATION

Each student needs a set of base-ten blocks and the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

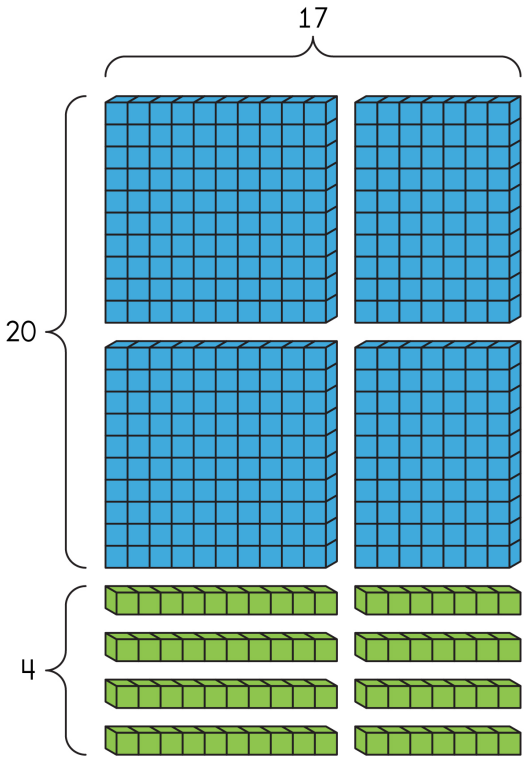
Require students to model with base-ten blocks the expression 24 × 17.

Ask students, "What does it mean to multiply two numbers? If I wanted to multiply 24 × 17, what does that mean? What if I wanted to multiply 17 × 24? Is there a difference between the two expressions?"

Emphasize that you can have either 24 groups of 17 or 17 groups of 24 and the total, or the product, will be equivalent.

Discuss that the property of operations that states the order of the factors in a multiplication expression does not change the value of the product is the *commutative property (of multiplication)*.

Model with base-ten blocks 20 groups of 17 *and* 4 groups of 17 (or 24 groups of 10 *and* 24 groups of 7). If you were to break apart the base-ten blocks, you would have 17 columns of 20 and 17 columns of 4. Students should be able to see both the 20 rows and the 4 rows each have 17 blocks. Each column is a group of 24 total blocks, therefore you have 24 groups of 17.



Explain that breaking numbers apart into expanded form is called *decomposing*.

Draw students' attention to the area model on Question 1 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT (24 × 17). **Ask** students to divide the area model according to the area model you made.

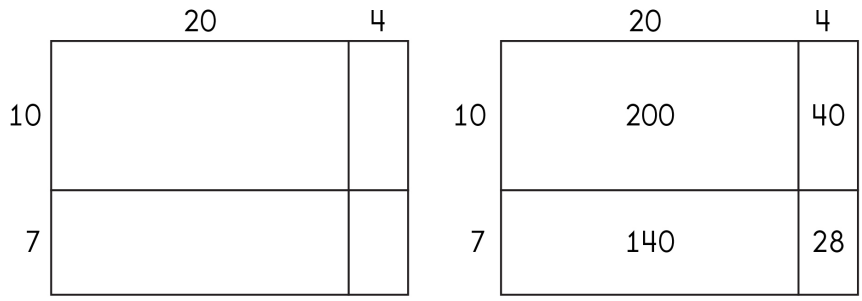
GUIDING QUESTIONS

Determine if the student can DECOMPOSE NUMBERS UP TO 1,000:

- When else have we seen or had to decompose numbers by place value?
- How could you decompose 74 in order to multiply it by nine easier?
- How could you decompose the number 24? 249? 2497?
- How could you decompose the factors 17 and 24?

Require students to write the decomposed numbers on the outside of the area model. Note that 17 and 24 can be decomposed either on the left hand side or the top of the model based on the commutative property.

Refer back to your model and have the students write the partial products inside the different sections of the area model.



Require students in partners or small groups (3 to 4 students) to use base-ten blocks to create an area model for Question 2 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students should count the number of blocks in the largest area, note that they have groups of tens, and realize they do not need to count each individual block to find the total.

Require students to write the number of blocks in the corresponding portion of the area model on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Next, **require** students to repeat the process with the other three sections of the area model.

Ask a different student to share each of the partial products with the class. They should share the decomposed numbers on the outside that they multiplied and the partial product.

GUIDING QUESTIONS

Determine if the student is ready to DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS:

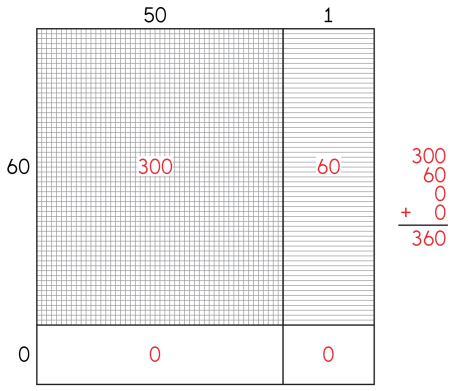
- What is a product? What is a partial product?
- Explain how to multiply 33 × 6 if you decompose 33 into 30 and three. What are the steps to calculate the product? How would you decompose 33 to multiply? What process would you follow once the number is decomposed?
- How would you put the partial products "back together" to find the total product?

Determine if the student can CALCULATE PRODUCTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM:

- Using these base-ten blocks, can you show me what one group of seven looks like? Now what does four groups of seven look like? How many blocks do you have with four groups of seven? How many blocks would you have if you had seven groups of four?

Model writing the four partial products and adding them to determine the total product on the right-hand side of the area model.

Emphasize that the digits must be lined up based on place value in order to add correctly. Each time you add the partial products, it is important to use the full value of the number. For example, students should add 300 and 60 if they are partial products rather than 3 and 6.



Require students to complete all the steps to find the product using partial products for Questions 3–5 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Require** one or two students to share with the class how they decomposed the numbers, the partial products, and the total product for each problem.

- 3. 48 × 92 = 4,416
- 4. 63 × 27 = 1,701
- 5. 59 × 24 = 1,416

Ask students, "Look at Question 3, 48 × 92. What do you notice about the numbers outside the area model and the numbers inside the area model?" (Students should comment on the fact that if you multiply the numbers on the outside of the area model, the numbers on the inside of the model are the partial products.)

Model Question 4, 63 × 27, on the board multiplying the decomposed factors on the outside of the area model and writing the partial product on the inside of the area model. When you multiply each of the decomposed factors, be sure to reference each factor by its full value. For example, 60 times 20 is 1200. This will help reinforce the importance of place value.

After determining all of the partial products, **write** the partial product addition on the right of the area model as modeled previously. **Emphasize** that the digits are all lined up according to place value to support the addition of the partial products.

Model decomposing 397 into 300 + 90 + 7 using base-ten blocks. Ask students, "Why would we break apart or decompose larger numbers when we multiply?" (Students should answer, "To make them easier to operate with.")

Explain, "Now that we know we can multiply the decomposed numbers to determine the partial products, we are going to multiply a three-digit number by a two-digit number."

Ask students:

- "How many columns is the area model divided into on Question 6?" (three)
- "Why are there three columns?" (930 is a three-digit number, and each column represents a place value: ones, tens, hundreds.)
- "How many rows is the area model divided into on Question 6?" (two)
- "Why are there two rows?" (59 is a two-digit number, and each row represents a place value: ones, tens.)
- "How many sections is the area model divided into on Question 6?" (six)
- "How are the number of sections on the area model and the number of digits in each divisor related?" (If you multiply the number of digits in the divisors by each other, the product is how many sub sections will be in the area model. For example, a two-digit × two-digit problem will have four sections.)
- "How will we decompose 59 and 930 to make more 'friendly' numbers to multiply?"

Emphasize that each column and each row of the area model is a place value. For example, the far-right column is the ones place, the next column to the left is the tens place, and the next column to the left is the hundreds place.

Model evaluating each of the partial products for Question 6. Once you have all of the partial products written to the right side of the area model, make explicit the number of partial products you have. This will be important when students no longer have the area model to visualize the number of partial products.

Emphasize that each of the partial products must line up according to place value in order to properly add the partial products.

Read the problem situations for Questions 7–9 before the students begin to set up and solve the problems using the area models.

- 7. Darren wants to save $70,000 to buy a house. If he saves $758 each month for 88 months, will he have saved enough money? (*No, Darren will only have saved $66,704.*)
- 8. Josh was researching and writing a paper on the American Revolution for his social studies class. Josh read 22 times more words than he wrote. His paper was 959 words long. How many words did he read to research the American Revolution? (*21,098 words*)
- 9. The middle school science teachers were taking a group of students to Washington D.C. for a science conference. 76 students were chosen to attend and the total cost per student for three days is $487. How much money will the science teachers need to collect for all 76 students to attend? (*The science teachers will need to collect $37,012.*)

GUIDING QUESTIONS

Determine if the student can EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION:

- Explain using base-ten blocks or a model of eight groups of 10 and eight groups of 100. How are the two problems the same? How are they different?
- If 62 represents six tens and two ones, and you have four groups of 62, how would you determine the product? If you had 40 groups of 62, how would you determine the product?
- Can you explain, using base-ten blocks to model, three groups of two? How does the problem change if you have three groups of 20? Three groups of 200?

Determine if the student can DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS:

- [Point to a student's partial products on Questions 7–9.] How did you determine these partial products?
- [Point to a student's work on Questions 7–9.] How did you decompose the factors to determine these partial products? Why did you do it that way?
- After you determine each of the partial products, what do you do to find the total product? How do you know?
- [Point to a student's work on Questions 7–9.] If you decomposed the factors differently, would you get the same partial products? How do you know? Would you get the same total product? How do you know?
- [Point to a factor on Questions 7–9.] What is another way you can decompose this factor? How do you know?
- [Point to a student's work on Questions 7–9.] Would decomposing either of these factors differently change your total product? How do you know?

Group students into partners and provide each pair with one stack of number cards from the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Require students to stack the number cards face down, like a deck of cards.

Explain that each partner will draw a number from the deck. The students will then set up an area model to multiply the two two-digit numbers on a sheet of paper or a dry erase board.

Once they finish, students should check their answer against their partner's answer. If there is a difference in the answers, challenge them to find the mistake. To scaffold the checking part, some students may check their work with a calculator to find the error.

Once students have finished solving and checking, they should place the numbers in a "used" pile and draw again. **Provide** students with enough time to complete at least five problems. To help the process, have them create problem situations for the numbers they have drawn in addition to determining the product.

GUIDING QUESTIONS

Determine if the student can DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS:

- [Point to student's partial products.] How did you determine these partial products?
- [Point to student's work.] How did you decompose the factors to determine these partial products? Why did you do it that way?
- After you determine each of the partial products, what do you do to find the total product? How do you know?
- [Point to student's work.] If you decomposed the factors differently would you get the same partial products? How do you know? Would you get the same total product? How do you know?
- [Point to a factor.] What is another way you can decompose this factor? How do you know?
- [Point to student's work.] Would decomposing either of these factors differently change your total product? How do you know?

Be sure to collect all sets of cards, as students will need them for the activity in LESSON 2.

At the end of the activity, teachers should require students to answer in pairs or small groups the following questions:

- What does it mean to decompose a number (for example, 429)?
- If you were to multiply 429 × 3, what would be the three expressions to find partial products?
- Once you have found all of the partial products, what do you need to do to determine the total product?
- Why is place value important when multiplying multi-digit whole numbers?

Name_____

Multiply Multi-Digit Whole Numbers

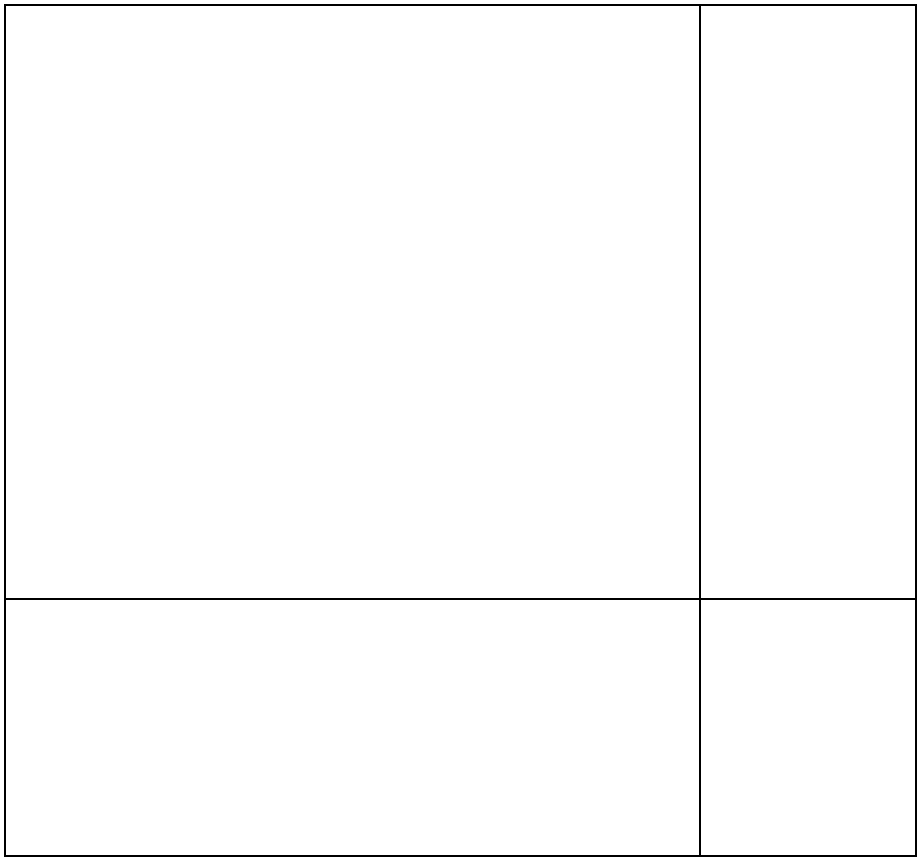
Lesson 1

1. 24 × 17

2. 51 × 60

Name_____

3. 48 × 92



4. Create an area model to solve: 63 × 27

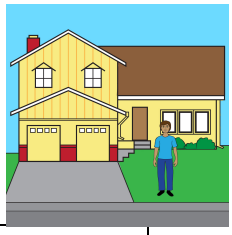
Name_____

5. Create an area model to solve: 59 × 24

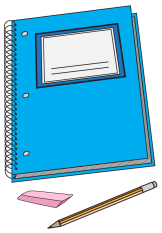
6. Create an area model to solve: 59 × 930

Name_____

7. Darren wants to save $70,000 to buy a house. If he saves $758 each month for 88 months, will he have saved enough money?



8. Josh and James submitted their final papers on the American Revolution for their social studies class. Josh wrote 82 times more words than James. James wrote 959 words. How many words long was Josh's paper? Create an area model to solve.



Name_____

9. The middle school science teachers were taking a group of students to Washington, D.C. for a science conference. 76 students were chosen to attend, and the total cost per student for three days is $487. How much money will the science teachers need to collect for all 76 students to attend? Create an area model to solve.

Multiply Multi-Digit Whole Numbers

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 1

12	13	14	15
16	17	18	19
23	24	25	26
27	28	29	32

34	35	36	37
38	39	43	45
46	47	48	49
52	53	54	56
57	58	59	63

64	65	67	68
69	73	74	75
76	78	79	83
84	85	86	87
89	94	97	98

MULTIPLY MULTI-DIGIT WHOLE NUMBERS

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will fluently multiply multi-digit whole numbers using the distributive property and partial products.

PRIMARY ACTIVITY

Students will use the "cluster" method by decomposing numbers and evaluating partial products to multiply multi-digit whole numbers.

NOTE: This lesson will likely require more than one class period to complete.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Factor
- Multiple
- Product
- Decompose/deconstruct
- Partial product
- Distributive property

MATERIALS

- Dry erase boards and markers or paper and pencils
- Number cards from Lesson 1 INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every two students.)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every two students.)
- Word Version INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- Word Version INSTRUCTIONAL ACTIVITY SUPPLEMENT

IMPLEMENTATION

Begin by writing the following expressions on the board for students to solve mentally (they should not use paper or dry erase boards):

12×1

12×10

12×100

$12 \times 1{,}000$

Ask a student to share the product of the first expression (*12*) with the class. **Write** the product next to the expression.

Note the identity property of multiplication (any number times one is the original number, e.g., $4 \times 1 = 4$ or $365 \times 1 = 365$), and since 12 is being multiplied by one, the product must be 12.

Proceed to the second expression, 12×10, and **require** a student to share the product and how they came to that answer (*120*) with the class. **Write** the product next to the expression.

Emphasize again the identity property, and this time include powers of 10. For example, *10 is 10 times more than one*. Therefore 120 is 10 times more than 12.

Require a student to share the product for the third expression (*1,200*) with the class. **Write** the product next to the expression.

Emphasize the identity property as well as the place value and power of ten relationships. "If 12 times one is 12, then 12 times 100 must be…" Do not state the pattern that is evolving; let the students discover the pattern on their own.

Require a student to share the product to the fourth expression (*12,000*) with the class. **Write** the product on the board next to the expression.

Ask students, "What do you notice about the expressions? Do you see any patterns? How are the expressions different?" The goal is for students to identify a relationship between the power of 10, the number of zeroes based on the exponent in the power of 10, and the number of zeroes in the product when the value is multiplied by another value. For example, 1,000 has three zeroes. If we multiply 12 times one, we get 12. If we multiply 12 times 1,000, we get 12 with three zeroes, which is 12,000 or 1,000 times more than 12.

NOTE: This discussion provides an opportunity to introduce or review the concept and pattern of multiplying (and dividing) by powers of ten

represented with exponents, which is addressed in standards 5.NBT.1 and 5.NBT.2. For example, think of or represent 12 × 1,000 as 12 × 10^3.

Use the following guiding questions to support or enhance discussion with students.

GUIDING QUESTIONS

Elicit student thinking:

- What does it mean to multiply two numbers?
- What patterns do you observe when you multiply two numbers?

Determine if the student can EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION:

- How is 12 × 10 different from 12 × 1,000? How are they similar?
- Is there a difference between multiplying 12 × 10 and 12 × 1,000? Explain.

Next, **write** the following expressions on the board, one at a time. Only write the next expression once you have written the product for the previous expression:

4 × 6

40 × 6

20 × 6

20 × 12

200 × 12

201 × 12

Require the students to determine the product mentally. **Select** one student to share the first product with the class (*24*). **Write** the product on the board next to the expression, then **write** the next expression on the board.

Require the students solve the second expression mentally, then have one student share the product (*240*) with the class. **Write** the product on the board next to the expression, then write the third expression on the board. Have students simplify the third expression mentally, then **require** one student to share the product (*120*) with the class.

Write the product on the board next to the expression and write the fourth expression on the board. **Allow** students to mentally determine the product, then require one student to share the answer (*1,200*) with the class.

Write the product on the board next to the expression and write the final expression on the board. **Allow** students to mentally determine the product, then have one student share the product (*1,212*) with the class. **Write** the product on the board next to the expression.

GUIDING QUESTIONS

Determine if the student can EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION:

- What is the identity property of multiplication? How does that help you multiply a number by 10? By 100?
- How are 4 × 6 and 40 × 6 the same? How are they different?
- What do you notice about 40 × 6 and 20 × 6? How are they different?
- How is 2 × 12 different from 200 × 12? How are they the same?
- How many times more than 20 × 12 is 200 × 12?
- Is 3 × 40 the same as 3 × 4? Why or why not?

Determine if the student is ready to MULTIPLY PAIRS OF NUMBERS WITH ANY NUMBER OF DIGITS:

- Explain why 40 × 6 and 20 × 12 have the same product.

Ask students about the similarities and differences among the problems. Use the previous guiding questions to support your discussion. **Wrap up** the discussion by asking students to compare the last two problems. Students should point out that the problems have a similar factor, 12, and that the other factor is increased by one, which can be related to the identity property of multiplication and that you simply need to add 12 to the product from the fifth problem.

Ask students, "What if I asked you to solve 201 × 12, but you didn't have all those other problems to help you. How else could you find the product?"

Require students to discuss their strategies with a partner, then have three or four students share their thinking with the whole class. Try to select students who utilize different strategies to multiply.

Write the students' ideas on the board to show their thinking; this can lead students with misconceptions to self-identify their mistakes or provide a way for you to redirect or correct the mistake in order to inform other students with the same misconception.

> NOTE: If you know there is a common mistake that is often made, you could present the incorrect work as "A student from last year/last class wrote this. What did the student do correctly? What did the student do incorrectly?"

Remind students that one way to multiply larger numbers is to break apart numbers to create more "friendly" numbers. Decomposing/deconstructing numbers strategically makes them easier to multiply. This results in partial products because we decomposed the problem into parts.

When added together, the partial products create the total product for the original expression. This process can also be called "clustering".

Write the expression 25 × 12 for students to consider.

Ask students, "How can we decompose these numbers to make 'friendlier' numbers?" Decompose the numbers according to place value, so that 25 becomes 20 and five, and 12 becomes 10 and two.

Write the following:

$$\begin{array}{r} 20 + 5 \\ \times\ 10 + 2 \\ \hline \end{array}$$

Connect students' understanding of the distributive property to decomposing and multiplying.

Direct students to use the distributive property to multiply, starting on the left with the largest numbers. Begin with 20 × 10 (*200*) and write the partial product directly under the expression (as you would with the standard algorithm).

Write the two factors next to the partial product so students can keep track of the different factor combinations.

$$\begin{array}{r} 20 + 5 \\ \times\ 10 + 2 \\ \hline 200 = 20 \times 10 \end{array}$$

Emphasize that because 20 and five are part of the same number, they should *not* be multiplied. The same is true for the 10 and the two. Linking this process to the distributive property will support students' conceptual understanding and minimize errors related to decomposition.

Require a student to share the product for 20 × 2 (*40*). This should be written directly underneath 200. **Emphasize** that the digits must be lined up based on place value.

Require a student to share the third partial product, 10 × 5 (*50*). This should be written directly underneath 40. Again, **emphasize** that the digits must be lined up based on place value.

Require a student to share the partial product of 5 × 2 (*10*). This should be written directly underneath 50 with the digits lined up based on place value.

Remind students that the numbers are lined up vertically based on place value. The ones column is on the right, the tens column is in the middle, and the hundreds column is on the left.

$$\begin{array}{r} 20 + 5 \\ \times\ 10 + 2 \\ \hline 200 = 20 \times 10 \\ 40 = 20 \times 2 \\ 50 = 10 \times 5 \\ 10 = 2 \times 5 \end{array}$$

Ask students, "Now that we have decomposed the original factors and used the distributive property to find partial products, what do we do to determine the total product for 25 × 12?" (*Add the partial products.*) As you add, ask students how and when to regroup. (*Regroup the 10 tens into 100, since 40 + 50 = 90, and 90 + 10 = 100. Therefore 25 × 12 = 300.*)

$$\begin{array}{r} 20 + 5 \\ \times\ 10 + 2 \\ \hline {\scriptstyle 1}200 = 20 \times 10 \\ 40 = 20 \times 2 \\ 50 = 10 \times 5 \\ +\quad 10 = 2 \times 5 \\ \hline 300 \end{array}$$

Emphasize that you do not multiply the values that were decomposed from the same original factor. This is a misconception that students may hold and needs to be addressed early in instruction.

Note that if there is an addition sign between the two digits, they should not be multiplied; they are factors that were broken into two parts. For example, 25 is *not* 20 × 5 but it *is* 20 + 5.

Share with students the following problem situation:

> Mrs. Smith plans to use 2,000 pencils in her class this year. There are 35 students in Mrs. Smith's fifth grade class. At the beginning of the year, each

> student brought 52 pencils to class. Will Mrs. Smith's class have enough pencils to use throughout the year?

Ask students what the multiplication expression would be for the problem situation and write the expression on the board. **Ask** students how they could decompose these numbers to make them "friendlier" to multiply. (*The 35 can become 30 and five, the 52 can become 50 and two*). **Write** the expression on the board with the decomposed numbers.

$$\begin{array}{r} 30 + 5 \\ \times\ 50 + 2 \\ \hline \end{array}$$

Require students to use the distributive property to determine and then share the partial products with the class, ensuring all four partial products are included and none are duplicated. **Emphasize** the fact that you are lining up the place values as you write the partial products on the board.

$$\begin{array}{rl} & 30 + 5 \\ \times & 50 + 2 \\ \hline & 1500 = 30 \times 50 \\ & 250 = 50 \times 5 \\ & 60 = 30 \times 2 \\ + & 10 = 5 \times 2 \\ \hline & 1820 \end{array}$$

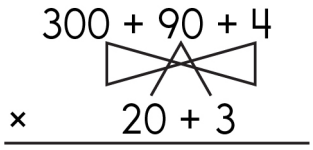
Reread the problem situation together with students to revisit what the question is asking, whether the product (*1,820 pencils*) addresses the question, and whether that is a reasonable answer. Student should respond that you need to compare the 1,820 pencils to the 2,000 pencils that Mrs. Smith will require.

Share with students the following problem situation:

> Matt wants to drive around all 48 contiguous states. He plans to drive 394 miles every day. So far he has driven for 23 days. How many miles has Matt traveled?

Ask students what the multiplication expression would be for the problem situation and write the expression on the board. As you solve, have a student come up to write on the board for each partial product.

Ask students, "How can we break apart the two factors into 'friendlier' numbers?" (*394 = 300 + 90 + 4 and 20 + 3*)



Require students who write a partial product on the board to also write the two factors for the partial product. **Emphasize** that this helps the students keep track of which factors have been multiplied so there are not any repeats. Eventually, writing the factors to the right of the partial product can be eliminated as students no longer need that scaffold.

Emphasize each time a student writes a partial product on the board that they make sure the digits are lined up according to place value.

Reread the problem situation providing the product: "If Matt drove 394 miles every day and he has driven for 23 days, he would have driven 9,062 miles so far on his road trip." **Ask** students whether the product answers the question being asked, whether it makes sense, and whether it is reasonable. Students should respond that it does answer the question and seems reasonable based on the values in the problem situation.

GUIDING QUESTIONS

Determine if the student can MULTIPLY PAIRS OF NUMBERS WITH ANY NUMBER OF DIGITS:

- If you were to multiply two two-digit numbers, how many partial products would you have? How many partial products would you have if you multiply a two-digit number by a three-digit number?
- When you multiply 6 × 245, you could multiply six by what three whole numbers?
- Explain why 24 × 3 is not 3 × 4 plus 3 × 2.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Read aloud the first problem situation on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

1. Mitchell's yard is 36 yards long and 23 yards wide. Mitchell has to mow the lawn every Saturday morning at 9:00 to earn his allowance. How many square yards of grass does Mitchell have to mow? (828 square yards)

Require students to complete the organizer on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT for the problem as you go through each of the steps.

Ask students, "How can we decompose 36 and 23 into 'friendlier' numbers?" (*30 and six, 20 and three*)

Require a student to share the first decomposed factors and the partial product. **Require** each student to write the partial product on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT page.

Repeat the process for each of the four partial products. **Require** the students to write the answer to the problem situation in a complete sentence.

Emphasize that when adding to determine the total product, the partial products must be lined up according to place value.

Repeat the aforementioned procedure for Questions 2 – 5 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Be sure to read aloud the problem situation for each problem, and require students to write the answer in a complete sentence.

2. Mia and Ava are going on a road trip around the United States. Their goal is to visit every state. So far Mia and Ava have driven for 43 hours. If they were driving 68 miles per hour, how many miles have Mia and Ava traveled so far? (*43 × 68 = 2,924 miles*)

3. Benjamin and Jose are building a deck. Benjamin thinks they will need 30,000 nails to complete the project. They bought 91 packages of nails, and each package contains 342 nails. Will Benjamin and Jose have enough nails to complete the project? (*Yes. 91 × 342 = 31,122*)

4. Hometown Hamburger was ordering buns for the restaurant for the following week. The manager ordered 39 cases of buns, and each case contains 696 hamburger buns. How many buns will Hometown Hamburger have for the next week? (*39 × 696 = 27,144 buns*)

5. Abby's video game has 76 character options. There are 487 different clothing options. How many different character and clothing combinations can Abby create? (*487 × 76 = 37,012*)

Arrange students into partners and give each pair two stacks of number cards (one from the Lesson 1 INSTRUCTIONAL ACTIVITY SUPPLEMENT and one stack from the Lesson 2 INSTRUCTIONAL ACTIVITY SUPPLEMENT). The students should keep the two stacks separate; one stack is two-digit numbers and the other stack is three-digit numbers.

Require students to stack the number cards face down, like a deck of cards.

Explain that each partner will draw a number from one of the decks; one student draws from the two-digit stack and one student draws from the three-digit stack. The students will then determine the product on a sheet of paper or dry erase board.

Once they finish, students should check their answer against their partner's answer. If there is a difference in their answers, challenge them to find the mistake.

Once students have finished checking, they should place the numbers in a "used" pile and draw again (have them create two "used" piles so the two-digit and three-digit cards do not get mixed up). Provide students with enough time to complete at least five problems.

At the end of the activity, teachers should require students simplify the following expressions as an exit ticket:

1. 55×92

CORRECT ANSWER

$50 \times 90 = 4{,}500$

$50 \times 2 = 100$

$5 \times 90 = 450$

$5 \times 2 = 10$

Total product = 5,060

COMMON MISTAKES AND MISCONCEPTIONS

Example Error	Misconception	Missing Knowledge
$5 \times 9 = 45$ **$5 \times 2 = 10$** **$9 \times 5 = 45$** **$5 \times 2 = 10$** **Total = 110**	multiplying the digits and not the values of each number represented, e.g., 5×9 instead of 50×90	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION
Leaves out a set of numbers/partial product.	does not determine all partial products	CALCULATE PRODUCTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM
50×5 ***or*** **90×2**	multiplies the two values of the same decomposed whole number together	CALCULATE PRODUCTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM
Student miscalculates a partial product. **e.g., $50 \times 90 = 4{,}000$**	does not multiply correctly	MULTIPLY PAIRS OF NUMBERS WITH ANY NUMBER OF DIGITS

2. 354 × 28

CORRECT ANSWER

300 × 20 = 6,000
300 × 8 = 2,400
50 × 20 = 1,000
50 × 8 = 400
4 × 20 = 80
4 × 8 = 32
Total Product = 9,912

COMMON MISTAKES AND MISCONCEPTIONS

Example Error	Misconception	Missing Knowledge
3 × 2 = 6 **3 × 8 = 24** **5 × 2 = 10** **5 × 8 = 40** **4 × 2 = 8** **4 × 8 = 32** **Total = 120**	multiplying the digits and not the values of each number represented, e.g., 3 × 2 instead of 300 × 20	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION
Student leaves out a set of numbers/partial product.	does not determine all partial products	CALCULATE PRODUCTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM
300 × 50 **300 × 4** **50 × 4** *or* **20 × 8**	multiplies the two values of the same decomposed whole number together	CALCULATE PRODUCTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM
Student miscalculates a partial product. **e.g., 4 × 8 = 36**	does not multiply correctly	MULTIPLY PAIRS OF NUMBERS WITH ANY NUMBER OF DIGITS

Name_____

Multiply Multi-Digit Whole Numbers

Lesson 2

1. Mitchell's yard is 36 yards long and 23 yards wide. Mitchell has to mow the lawn every Saturday morning at 9:00 to earn his allowance. How many square yards of grass does Mitchell have to mow?

36 = _____ + _____

23 = _____ + _____

_____

_____

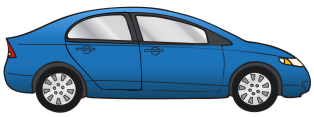
_____

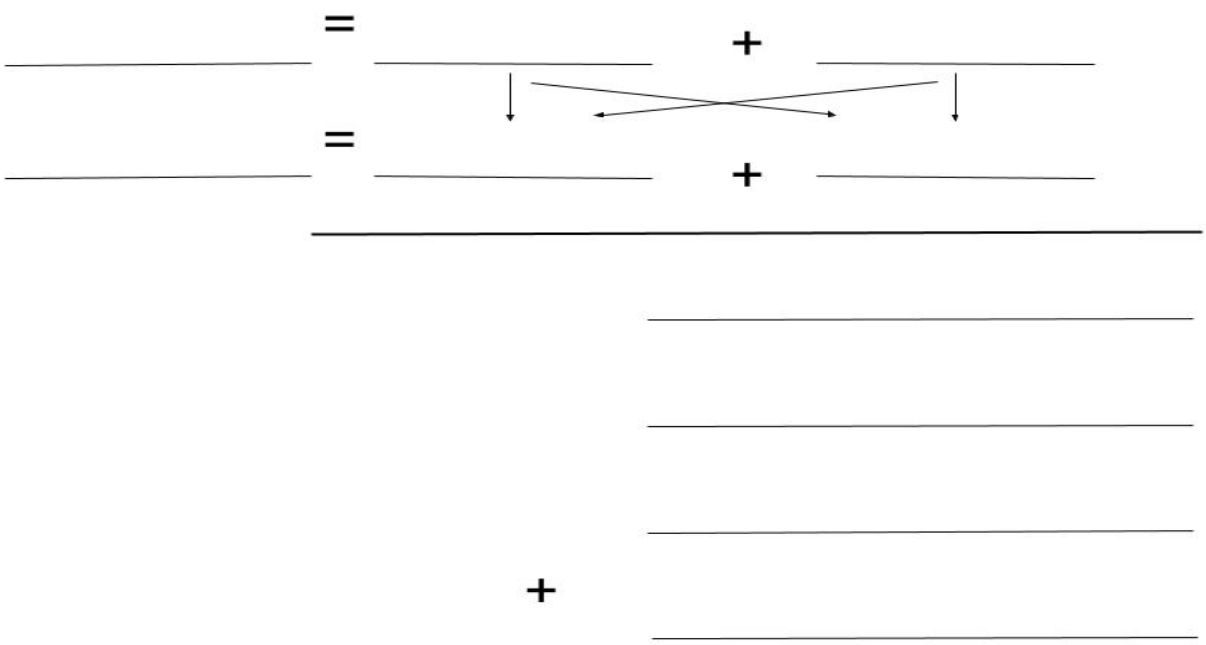
_____

\+ _____

Name_____

2. Mia and Ava are going on a road trip around the United States. Their goal is to visit every state. So far Mia and Ava have driven for 43 hours. If they were driving 68 miles per hour, how many miles have Mia and Ave traveled so far?





Name_____

3. Benjamin and Jose are building a deck. Benjamin thinks they will need 30,000 nails to complete the project. They bought 91 packages of nails, and each package contains 342 nails. Will Benjamin and Jose have enough nails to complete the project?

342 = _____ + _____ + _____

91 = _____ + _____

_____

_____

_____

_____

_____

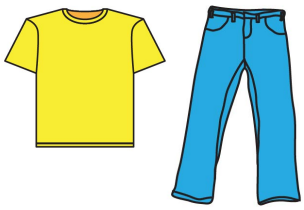
\+ _____

Name_____

4. Hometown Hamburger was ordering buns for the restaurant for the following week. The manager ordered 39 cases of buns, and each case contains 696 hamburger buns. How many buns will Hometown Hamburger have for the next week?

Name_____

5. Abby's video game has 76 character options. There are 487 different clothing options. How many different character and clothing combinations can Abby create?



Multiply Multi-Digit Whole Numbers

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2

103	207	325	426
531	643	728	831
934	139	245	347
459	555	672	764

863	978	182	285
391	497	503	614
715	823	925	137
239	342	458	561
664	769	873	976

185	287	392	496
503	608	714	819
921	135	238	342
449	563	657	754
861	976	384	792

MULTIPLY MULTI-DIGIT WHOLE NUMBERS

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will fluently evaluate multi-digit multiplication using the standard algorithm.

PRIMARY ACTIVITY

Students will use the standard algorithm to multiply multi-digit whole numbers.

NOTE: This lesson will likely require more than one class period to complete.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Factor
- Multiple
- Product
- Decompose/deconstruct
- Partial product
- Regroup
- Distributive property

MATERIALS

- Dry erase boards
- Dry erase markers
- Lined paper
- Pencils
- Paper lunch bags: three per partner group, one labeled "Hundreds", one labeled "Tens", and one labeled "Ones"

- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every two students.)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- Word Version INSTRUCTIONAL ACTIVITY SUPPLEMENT
- Word Version INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

IMPLEMENTATION

Distribute dry erase boards and markers or plain paper and pencils.

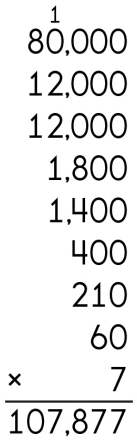
Write the expression 74 × 26 (*1,924*) on the board.

Ask students, "How would you simplify this expression?" **Require** students to simplify on dry erase boards or paper, using an area model or cluster method. Select one or two students to share with the class how they simplified the expression, explaining each step.

Repeat the process with 415 × 15 (*6,225*).

Ask students, "What if you were going to multiply 467 × 231? How many partial products would there be?" **Explain** that finding nine partial products could be challenging to keep in order. **Model** on the board an area model with the partial products off to the right, to show students how much writing these strategies would involve.

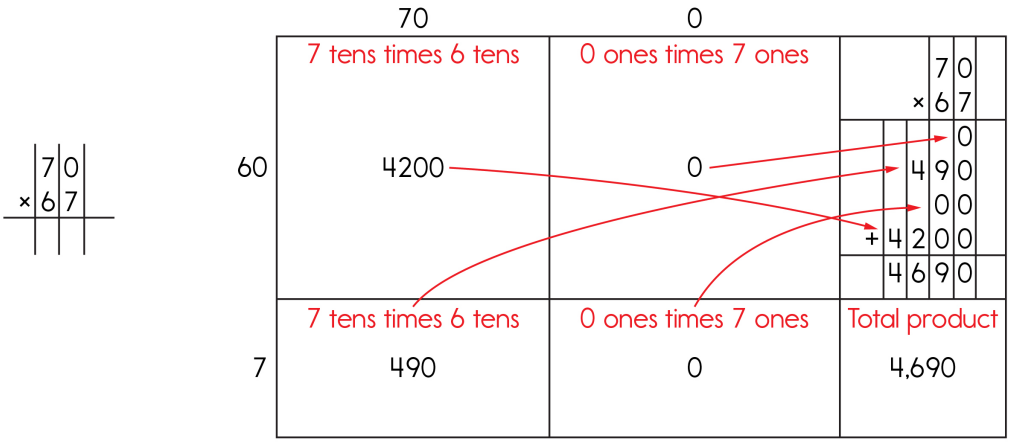
	400	60	7
200	80,000	12,000	1,400
30	12,000	1,800	210
1	400	60	7



Explain that there is a method that allows you to multiply large numbers more efficiently.

Write 70 × 67 so that the expression is stacked vertically. Draw lines so that it looks like what the students will have on their paper. Also, create an area model to help students see the transition from area model/partial products to the standard algorithm.

> NOTE: The connection between the use of the distributive property with partial products and the standard algorithm may require extended time and practice for students to develop fully.



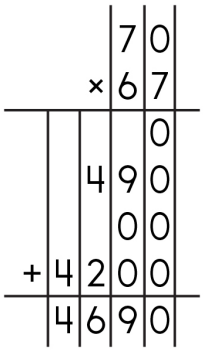
Explain that you are multiplying seven tens (*70*) times six tens (*60*) and seven ones (*7*). **Emphasize** that each column represents a place value: ones, tens, hundreds, and so on.

Remind students that when they multiplied using partial products, they decomposed the numbers and multiplied according to place value. Start with the ones place value and multiply 7 × 0. Write the zero directly underneath the problem in the ones column.

Explain, "Now we are going to multiply 7 × 70 (*490*). The four goes in the hundreds column for four hundreds, the nine goes in the tens column for nine tens, and the zero goes in the ones column."

Repeat the process for 60 × 0 and 60 × 70. As you solve each partial product, be sure to use base-ten language: six tens and seven tens, or 60 and 70. Do not refer only to the digits.

Emphasize the place value of the partial products each time. Once you have all the partial products, **remind** students that to find the total product, you need to add together all the partial products.



NOTE: It does not matter in what order students write the partial products. The emphasis is on the place value of the numbers. The partial products can be written in any order. Some students may work the larger values first.

Evaluate 568 × 51 (*28,968*) using the same procedure, requiring students to write the partial products after you demonstrate on the board. Each time you multiply a partial product, have the students share the product with the class, either by calling out together or calling on an individual student. For example, you would say, "1 × 8 is…" and the student or students would respond "eight".

Repeat the process to evaluate 832 × 82 (*68,224*). Have a student come up to the board for each partial product to write on the board. Always have students write the partial products on their papers after the board. Each time, **emphasize** the place value of the factors and the partial products.

Have students work in pairs or independently to complete Questions 1 – 5 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. This will give you time to work one-on-one or in a small group with students, or to walk around to monitor progress. Use the guiding questions to scaffold student thinking.

1. 82 × 23 = 1,886
2. 45 × 24 = 1,080
3. 97 × 49 = 4,753
4. 696 × 39 = 27,144
5. 487 × 76 = 37,012

GUIDING QUESTIONS

Elicit student thinking:

- What is multiplication?
- Can you model or explain what 2 × 3 means?

Determine if the student can EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION:

- If you decompose 49, how many tens do you have? How many ones?
- Is multiplying 40 × 7 the same as multiplying 4 × 7? Why or why not?
- Is the partial product 64 the same as 640? Explain how they are the same or different.
- What place value is the four in 46? So, if you multiply 46 × 12, are you multiplying 2 × 4 or 2 × 40?

Bring the class back together and have students share their answers to Questions 1 – 5 with the class. Have a student share how they found the answer for each problem. Be sure to address any misconceptions.

Explain, "When you multiplied the larger whole numbers, did you notice that you still have many partial products to add? What if we could solve the product even more efficiently?" Write 56 × 42 vertically, with lines to separate digits based on place value as in the previous example. Write the expression twice on the board. The first expression will be used to show the previous method, and the second will be used to show regrouping.

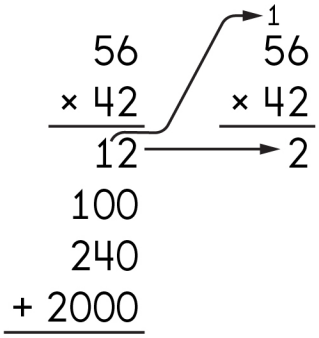
Model how to simplify 56 × 42 using the method from the beginning of the lesson, writing all partial products under the factors.

$$\begin{array}{r} 56 \\ \times\ 42 \\ \hline 12 \\ 100 \\ 240 \\ +\ 2000 \\ \hline \end{array}$$

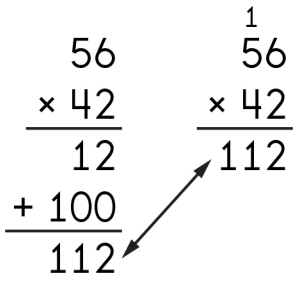
Next, students will simplify the same expression using the standard algorithm and regrouping.

Explain that instead of writing the partial product underneath, you can regroup and write the carried value above the existing value in the expression. Then add the carried value to the new partial product as you would with the partial products underneath the expression. For example, with 2 × 6 = 12, the "2" in "12" would go underneath the problem in the ones place, and because you have one ten to regroup, the "1" would carry to the tens column above the five in "56".

When you multiply 2 × 50 = 100, add in the one ten for 110.



Emphasize the importance of place value. With the standard algorithm, you would represent 110 by writing a "1" in the tens column and a "1" in the hundreds column; the "2" from 2 × 6 = 12 remains in the ones column since 110 has zero ones. **Show** students that when you add the two partial products, it is equal to what is written underneath the problem with the standard algorithm. (12 + 100 = 112 is the same as 110 + 2)



Draw students' attention to the partial products in the first model to show the comparison with the standard algorithm where regrouping occurs above the factors.

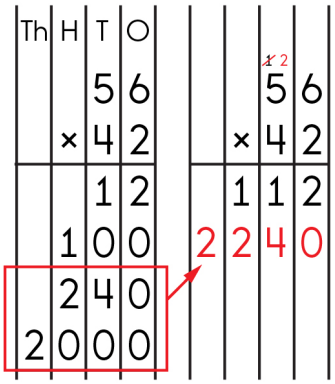
State that you are regrouping a number into the proper place value above the factors to be more efficient (less writing, less paper).

Continue to **emphasize** place value as students multiply. For example, **remind** students that you are multiplying 2 × 50 rather than 2 × 5.

Repeat the same process with 40 × 6 and 40 × 50. 40 × 6 = 240, therefore the 40 would go underneath the problem because you are now working from the tens place, and the "2" would carry over to the hundreds place. When you multiply 40 × 50 = 2,000, you will need to add in the carried value (*200*) for a total of 2,200. Because these two partial products are combined under the problem in the standard algorithm, you have 2,200 + 40 = 2,240.

Remind students when regrouping, you must add in the value that has been regrouped. The lines serve to distinguish the digits by place value, so they are not regrouping "2", they are regrouping "20".

As in the previous lessons, **refer** to the total value of the number when you explain your steps. For example, use 2 × 50, 40 × 6, and 40 × 50.



Have students return to their lined paper, or get a new sheet if needed.

Repeat the process for 150 × 882, requiring students to write the partial products after you demonstrate on the board.

Each time you multiply a partial product, have the students share the product with the class, either by calling out together or calling on an individual student. For example, you would say, "8 × 0 is…" and the student or students would respond "zero".

Emphasize the place value of regrouping and partial products each time you multiply.

When you write the partial products under the problem, **make explicit** that the place values carry down. You would not multiply 50 × 2 and represent the product in tens; it would be 100, therefore the partial products should be written in the correct place value.

GUIDING QUESTIONS

Determine if the student can MULTIPLY A 2-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING:

- What does it mean to regroup?
- When have we used regrouping before?
- When you multiply 6 × 2 from 56 × 42, how many ones do you have? How many tens? (2 ones, 1 ten)
- What other values do you already have in the tens place? (5 tens or 50, and 4 tens or 40)
- How would you regroup the one ten from 12 into the total product?
- When you multiply 40 × 6 from 56 × 42, how many ones do you have? How many tens do you have? How many hundreds do you have? (zero ones, four tens, two hundreds)
- How would you regroup the two hundreds from the 240 into the total product?

Introduce the next two problems as real life situations before you begin simplifying the expressions on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Repeat the previous steps to multiply. After solving each problem situation, reread the problem aloud. Make sure the product answers the question, and that the answer makes sense and is reasonable.

6. Janie and her friends are setting up an obstacle course for their school's Fun Run. In order for there to be enough room for all of the obstacles, they need at least 17,000 square feet. The host blocked off a space that was 72 feet wide and 244 feet long. Will that be enough room for Janie and her friends to set up their obstacles? (*Yes, it is 17,568 square feet.*)

7. Greg was researching fire ants for a research project. He read that in a remote area of Africa, there are 593 different fire ant colonies. If each colony has a population of 944 ants, what should Greg state is the total population of fire ants in that remote area of Africa? (*559,792 fire ants*)

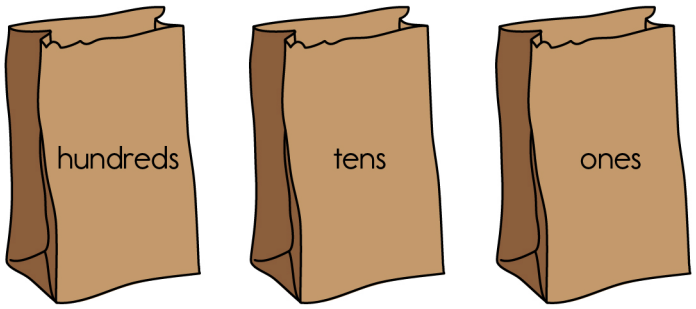
GUIDING QUESTIONS

Determine if the student can MULTIPLY A 3-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING:

- If you multiply 7 tens × 4 tens, how many tens do you have? How many hundreds do you have? What number did you have to regroup to the hundreds place?
- If you multiply 7 tens × 2 hundreds, how many hundreds do you have? How many thousands do you have? How many ten-thousands do you have? What value did you have to regroup to the thousands place? What value did you have to regroup to the ten-thousands place?
- How is regrouping when multiplying a three-digit number by a two-digit number similar to regrouping when multiplying a two-digit number by a two-digit number?

Group students into pairs. Each pair needs three paper lunch bags and the number cards from the INSTRUCTIONAL ACTIVITY SUPPLEMENT. The cards will need to be divided up by place value: all the hundreds together, all the tens together, and all the ones together.

Require students to place the cards into the paper bags with the corresponding label. The hundreds cards go into the hundreds bag, the tens cards go into the tens bag, and the ones go into the ones bag.



Explain to the students that each partner will draw a two- or three-digit number from the bag.

Because the cards are separated based on place value, the students will have to write the numbers in standard form before they multiply. For example, drawing 400, 60, and 3 would be 463.

Once students multiply, they should check their answer with their partner. If there is a difference in their answers, challenge them to find the mistake.

Once students have finished solving and checking, they should place the numbers back into the bags, shake them up, and draw again.

Provide students with enough time to complete at least five problems: two three-digit by two-digit problems, and three three-digit by three-digit problems.

Students should be required eventually to use the standard algorithm to solve multi-digit multiplication problems.

At the end of the activity, teachers should display the following questions and have students discuss their responses with a partner. **Require** one or two groups to share with the whole group.

- Jane simplified 182 × 57. Is Jane correct? If so, explain why. If not, describe her error and provide the correct answer. (*The reasoning to convince Jane should be that the mistake Jane made was she multiplied by 50. She is off by a place value; she actually multiplied by five. Instead of multiplying 50 × 2, 50 × 80, and 50 × 100, she did 5 × 2, 5 × 80, and 5 × 100. The correct answer is 10,374.*)

$$\begin{array}{r} {\scriptstyle 84\ 21} \\ 182 \\ \times\ 57 \\ \hline {\scriptstyle 1}1274 \\ +\ 910 \\ \hline 2{,}184 \end{array}$$

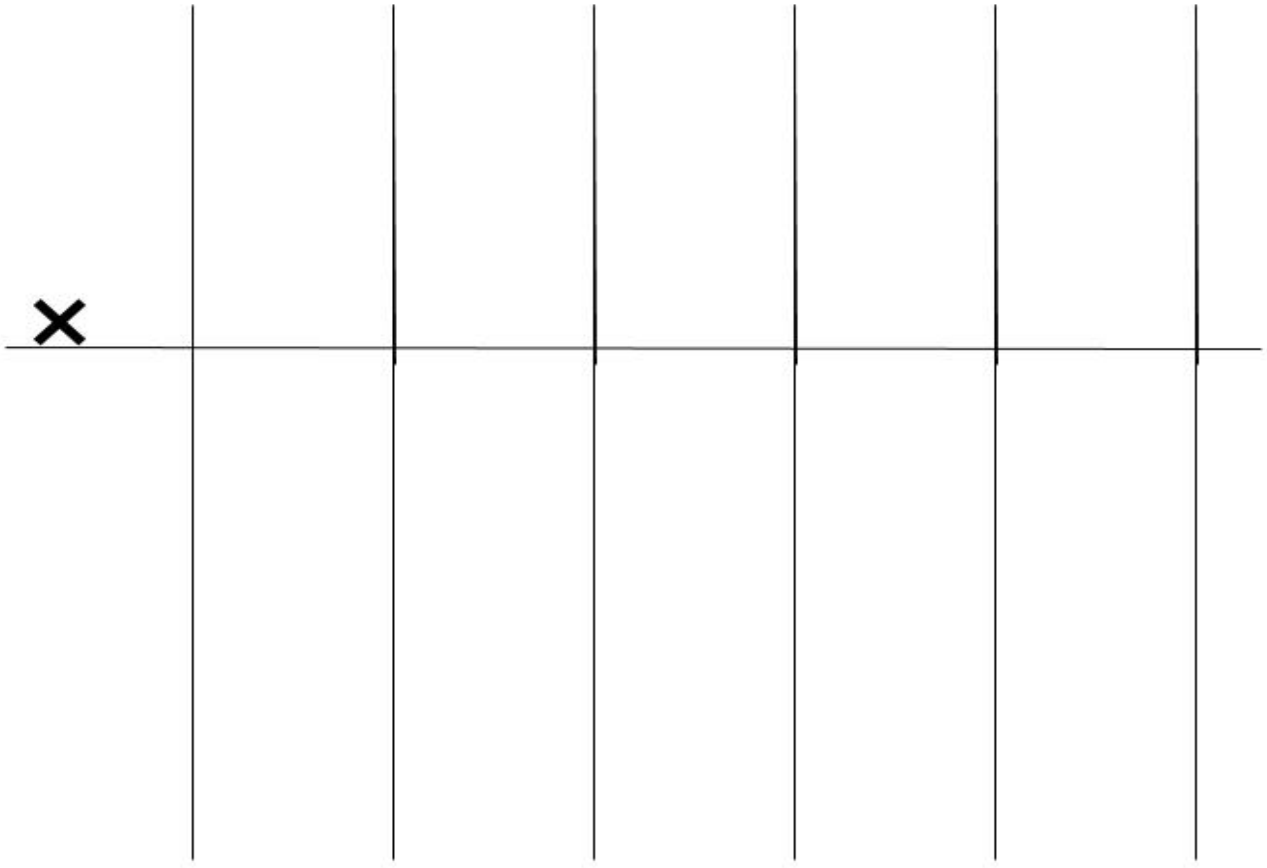
- Explain how to simplify 395 × 51 using the standard algorithm. (*Verify student answers for accuracy.*)

Name_____

Multiply Multi-Digit Whole Numbers

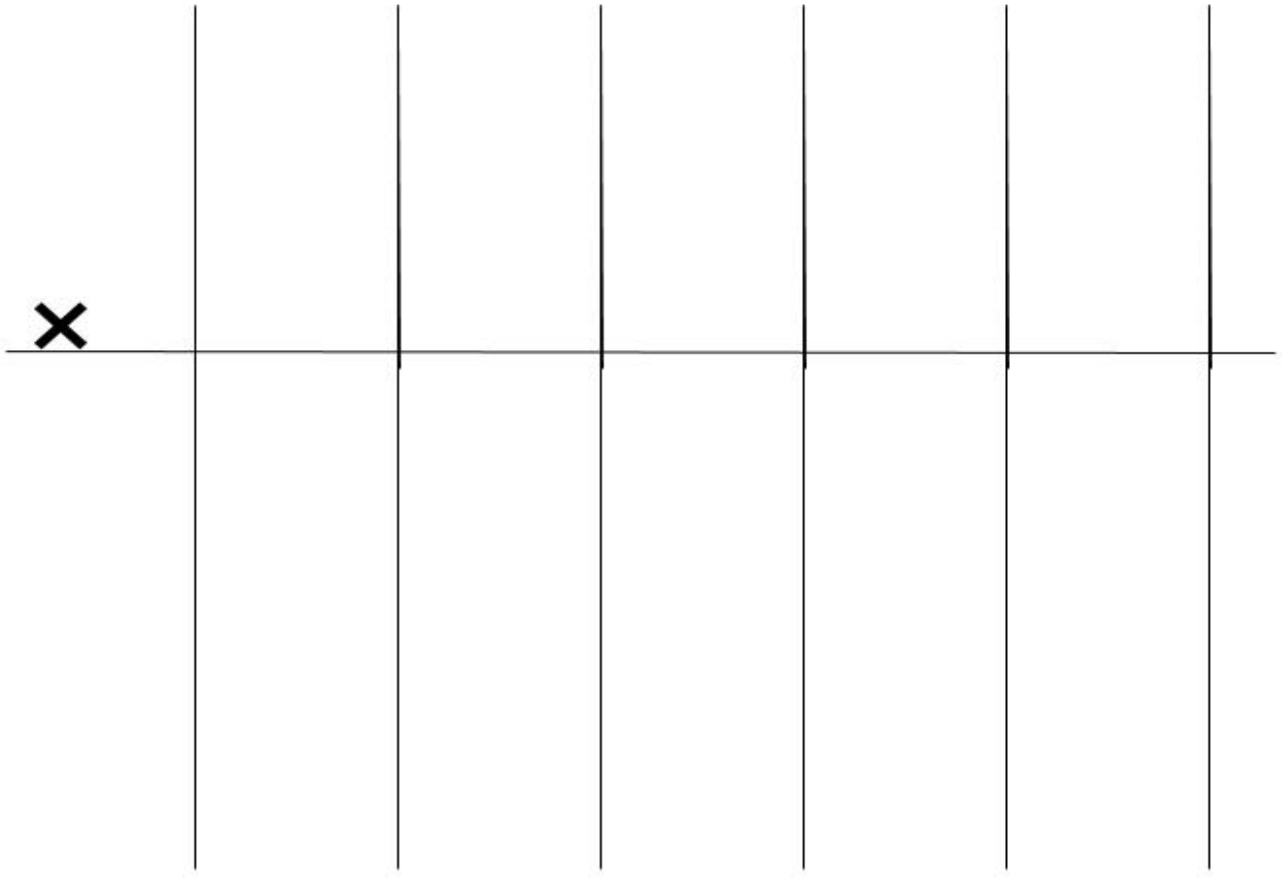
Lesson 3

1. 82 × 23



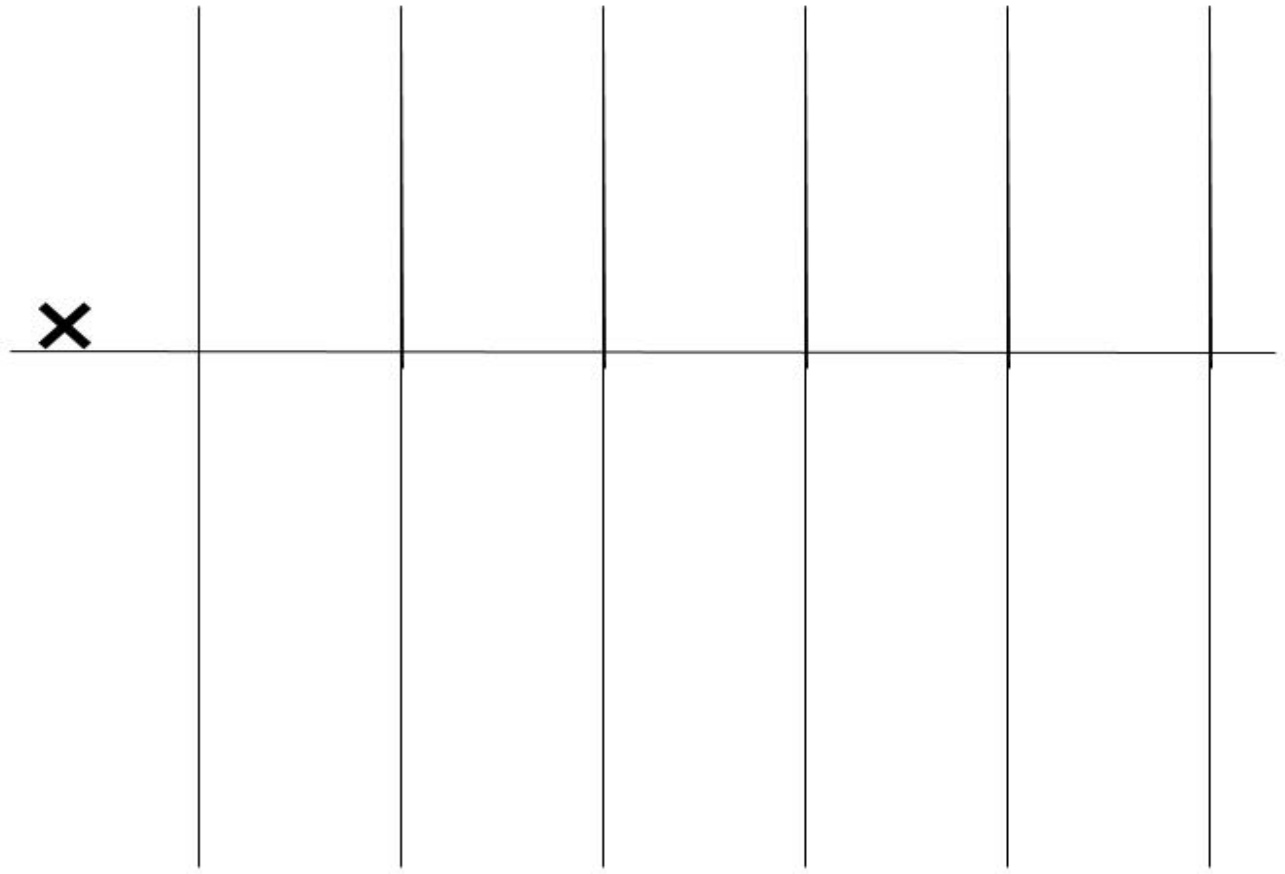
Name_____

2. 45 × 24



Name_____

3. 97 × 49



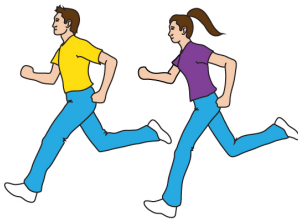
Name_____

4. 696 × 39

5. 487 × 76

Name_____

6. Janie and her friends are setting up an obstacle course for their school's Fun Run. In order for there to be enough room for all of the obstacles, they need at least 17,000 square feet. The host blocked off a space that was 72 feet wide and 244 feet long. Will that be enough room for Janie and her friends to set up their obstacles?



7. Greg was researching fire ants for a research project. He read that in a remote area of Africa, there are 593 different fire ant colonies. If each colony has a population of 944 ants, what should Greg state is the total population of fire ants in that remote area of Africa?

Multiply Multi-Digit Whole Numbers

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 3

1	2	3	4
5	6	7	8
9	1	2	3
4	5	6	7

8	9	10	20
30	40	50	60
70	80	90	10
20	30	40	50
60	70	80	90

100	200	300	400
500	600	700	800
900	100	200	300
400	500	600	700
800	900		

Name_____

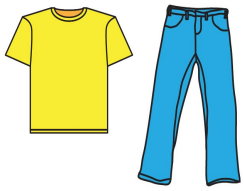
Multiply Multi-Digit Whole Numbers

1. Solve the following situations using an area model.

1.a. Monica plays her online game 52 minutes a month. Devon plays his online game 43 times more often a month than Monica. How many minutes does Devon spend playing his online game a month?

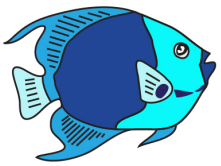
Name_____

1.b. Maggie and Mackenzie were shopping for new school clothes at Always 18. The store had 838 different tops to choose from and 26 different pant or shorts options. How many clothing combinations could Maggie and Mackenzie possibly make with all the clothes at Always 18?



Name_____

1.c. The City Aquarium has 36 different fish exhibits. Each exhibit holds 458 fish that need to be fed every day. How many fish does the aquarium staff feed every day?



1.d. Complete the area model, then determine the product.

	100		2
		1600	
7			

Name_____

1.e. Will responded that the "?" is equal to 2. Is Will correct? If so, explain how you know. If not, explain the correct answer and why Will is incorrect.

	600	?	5
10	6000	200	50
8	4800	160	40

Name_____

2. Jack and Jill completed the following partial products for the problem 349 × 63.

Jack's Work

$$
\begin{array}{r}
300 + 40 + 9 \\
\times \quad 60 + 3 \\
\hline
18000 = 300 \times 60 \\
2400 = 60 \times 40 \\
560 = 60 \times 9 \\
9 = 300 \times 3 \\
12 = 40 \times 3 \\
+ \quad 27 = 9 \times 3 \\
\hline
21{,}008
\end{array}
$$

Jill's Work

$$
\begin{array}{r}
300 + 40 + 9 \\
\times \quad 60 + 3 \\
\hline
18000 = 300 \times 60 \\
900 = 300 \times 3 \\
120 = 40 \times 3 \\
12000 = 300 \times 40 \\
2400 = 60 \times 40 \\
27 = 9 \times 3 \\
+ \\
\hline
33{,}447
\end{array}
$$

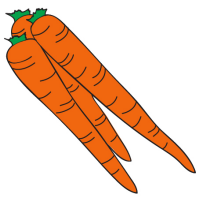
2.a. Is Jack's work correct? If yes, explain how you know. If not, explain what Jack did that is incorrect.

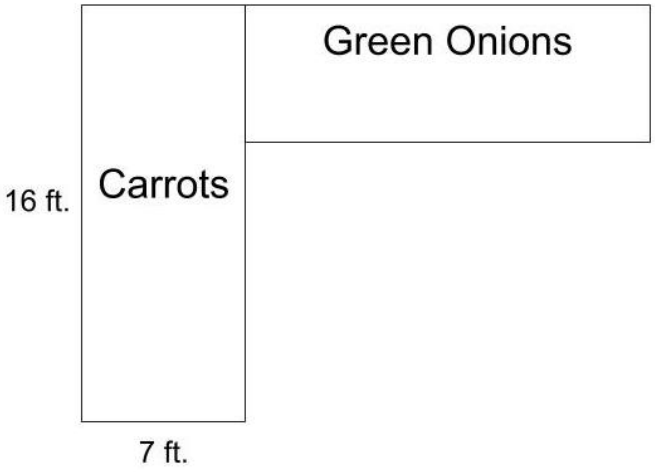
2.b. Is Jill's work correct? If yes, explain how you know. If not, explain what Jill did that is incorrect.

Name_____

2.c. Use the distributive property to solve the problem.

3. Jules was helping her grandmother prep and plant part of her garden. Her grandmother wanted to plant green onions and carrots. Use the information in the image to help you answer the following questions.





Name_____

3.a. What is the total area of the carrot section in square feet?

_____ + _____

_____

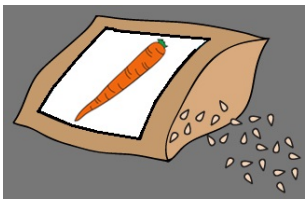
× _____

_____ = _____ × _____

_____ = _____ × _____

_____ = Total Product

3.b. A packet of carrot seeds contains 1,200 seeds. If Jules plants 16 carrot seeds per square foot, will one packet of seeds be enough?



Name_____

3.c. The area of the section for the green onions is 105 square feet and Jules's grandmother wants to plant 16 seeds in each square foot. A packet of green onion seeds contains 1,000 seeds. How many seeds does Jules's grandmother want to plant? How many packets of green onion seeds will Jules's grandmother need to purchase?

4. Describe the steps necessary to multiply a three-digit number by a two-digit number using the cluster method. Support your answer by creating and solving an example.

Name_____

5. Use the expression 576 × 54 to answer the following questions.

5.a. Decompose the factors in the expression.

5.b. Use the distributive property to determine the partial products for the expression.

5.c. What is the total product?

Name_____

6. Answer the following questions using the standard algorithm.

6.a. Chris has his own food truck. On Mondays and Tuesdays, he parks his truck in the city near the museums and office buildings. Chris usually sells 672 tacos when he parks in the city. On Fridays and Saturdays, he parks his truck near the beach. Chris sells 95 times more tacos on the weekend than he does on Mondays and Tuesdays. How many tacos does Chris sell during the weekends?

6.b. Marco wants to paint his game room blue. Three of his walls are the same size. Each wall is 295 inches long and 96 inches tall. How many square inches will Marco need to paint for all three walls?

Name_____

6.c. Macey is creating a 10-slide digital presentation for school. She has 361 different fonts available to use and 376 different color options. How many possible font and color combinations does Macey have to choose from when making her presentation?

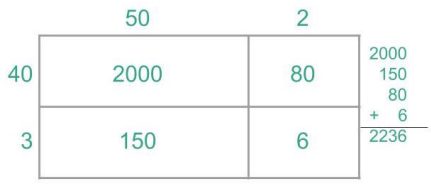
MULTIPLY MULTI-DIGIT WHOLE NUMBERS

STUDENT ACTIVITY SOLUTION GUIDE

1. Solve the following situations using an area model.

1.a. Monica plays her online game 52 minutes a month. Devon plays his online game 43 times more often a month than Monica. How many minutes does Devon spend playing his online game a month?

CORRECT ANSWER



Devon spends 2,236 minutes a month playing his online game.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not decompose the factors properly. For example, both 50 and 40 are on the top of the model and 2 and 3 are on the side of the model.	does not understand how to decompose numbers by place value	DECOMPOSE NUMBERS UP TO 1000
The student does not multiply the factors correctly based on place value. For example, $50 \times 40 \neq 200$.	does not understand that a tens value times a tens value equals a hundreds value	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION
The student is not clear on *what* the problem is asking.	does not recognize what the question is asking	SOLVE MULTIPLICATIVE COMPARISON PROBLEMS and USE REASONING FOR MULTIPLICATION AND DIVISION

1.b. Maggie and Mackenzie were shopping for new school clothes at Always 18. They were so overwhelmed by all the clothes. The store had 838 different tops to choose from and 26 different pant or shorts options. How many clothing combinations could Maggie and Mackenzie possibly make with all the clothes at Always 18?

CORRECT ANSWER

	800	30	8	
20	16000	600	160	16000 4800
6	4800	180	48	600 180 160 + 48 21788

Maggie and Mackenzie can make 21,788 different clothing combinations.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not decompose the factors properly. For example, both 800, 30, and 20 are on the top of the model, and 8 and 6 are on the side of the model.	does not understand how to decompose numbers by place value	DECOMPOSE NUMBERS UP TO 1000
The student does not multiply the factors correctly based on place value. For example, 30 × 20 ≠ 60.	does not understand that a tens value times a tens value equals a hundreds value	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION
The student is not clear on *what* the problem is asking.	does not recognize what the question is asking	SOLVE CARTESIAN PRODUCTS and USE REASONING FOR MULTIPLICATION AND DIVISION

1.c. The City Aquarium has 36 different fish exhibits. Each exhibit holds about 458 fish that need to be fed every day. How many fish does the aquarium staff feed every day?

CORRECT ANSWER

	400	50	8	
30	12000	1500	240	12000 2400
6	2400	300	48	1500 300 240 + 48 16488

The aquarium staff feeds 16,488 fish every day.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not decompose the factors properly. For example, both 400, 50, and 30 are on the top of the model, and 8 and 6 are on the side of the model.	does not understand how to decompose numbers by place value	DECOMPOSE NUMBERS UP TO 1000
The student does not multiply the factors correctly based on place value. For example, 50 × 6 ≠ 30.	does not recognize that 50 is 10 times more than 5, therefore the product of 6 × 50 is ten times greater than 6 × 5	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION
The student is not clear on *what* the problem is asking.	does not recognize what the question is asking	SOLVE PART/WHOLE PROBLEMS and USE REASONING FOR MULTIPLICATION AND DIVISION

1.d. Complete the area model then solve for the total product.

CORRECT ANSWER

	100	80	2	
20	2000	1600	40	2000 1600
7	700	560	14	700 560 40 + 14 4914

or

	100	40	2	
40	4000	1600	80	4000 1600
7	700	280	14	700 280 80 + 14 6674

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not write a tens value on the outside of the area model and instead uses a ones value or hundreds value.	does not understand how to decompose numbers by place value	DECOMPOSE NUMBERS UP TO 1000 and EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION
The student does not multiply the factors correctly based on place value.	does not understand that a tens value times a tens value equals a hundreds value	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION
The student does not complete the partial products that result from the unknown factors (the "?" on the outside of the area model).	unable to determine the missing factors given the available information	USE REASONING FOR MULTIPLICATION AND DIVISION and MULTIPLY A 2-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING
The student does not solve for the total product and only completes the area model.	may not understand that the partial products need to be added together for the total product because the factors were decomposed	DETERMINE THE PRODUCT OF TWO FACTORS USING PARTIAL PRODUCTS

1.e. Will responded that the "?" is equal to two. Is Will correct? If so, explain how you know. If not, explain the correct answer and why Will is incorrect.

	600	?	5
10	6000	200	50
8	4800	160	40

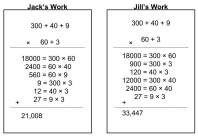
CORRECT ANSWER

The missing value is 20 because 200 is 10 times more than 20 and the given factor is 10. 2 times 10 would only be 20 and the given partial product is 200. Also, 8 times 20 is 160, which is 10 times more than 8 times 2, which is only 16.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student agrees with Will and thinks the missing value is 2.	does not recognize that 200 is the product of 10 times 20, which is 10 times greater than 20, the product of 10 times 2, or that 160 is the product of 8 times 20, which is ten times greater than 16, the product of 8 times 2	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION and MULTIPLY A 2-DIGIT NUMBER BY A 2-DIGIT NUMBER WITHOUT REGROUPING
The student provides the correct answer but cannot explain why that is correct or why Will is incorrect.	unable to communicate that the missing value should be 20, because it is ten times greater than 2 and the partial products are ten times greater than they would be if the factor was 2 instead of 20	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION and USE REASONING FOR MULTIPLICATION AND DIVISION

2. Jack and Jill completed the following partial products for the problem 349 × 63.



2.a. Is Jack's work correct? If yes, explain how you know. If not, explain what Jack did that is incorrect.

CORRECT ANSWER

> Jack did not multiply correctly when he multiplied 300 × 3 or 40 × 3. He did not consider the value of 300 or 40, just the digits 3 and 4.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student thinks that Jack's work is correct.	does not recognize the place value of the factors being multiplied and just multiplies the digits	MULTIPLY ONE-DIGIT WHOLE NUMBERS BY MULTIPLES OF 10
The student identifies that Jack is incorrect but cannot explain what he did wrong.	unable to communicate the errors	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION and DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS

2.b. Is Jill's work correct? If yes, explain how you know. If not, explain what Jill did that is incorrect.

CORRECT ANSWER

Jill did not multiply correctly. She multiplied two values that are part of the same decomposed number. She multiplied 300 × 40, which is incorrect because they are both decomposed parts of 349. Also, Jill did not multiply 9 × 60.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student thinks that Jill's work is correct.	does not recognize which values to multiply when two larger numbers have been decomposed	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION and DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS
The student identifies that Jill is incorrect, but cannot explain what she did wrong.	unable to communicate the errors	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION and DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS

2.c. Use partial products to solve the problem.

CORRECT ANSWER

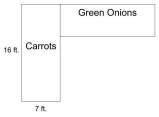
```
      300 + 40 + 9
  x       60 + 3
-------------------
 1800 = 300 × 60
  900 = 300 × 3
 2400 = 40 × 60
  120 = 40 × 3
  540 = 9 × 60
+  27 = 9 × 3
-------------------
21987 = Total Product
```

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student is missing partial products or multiplies two values from the same decomposed number.	may not understand that a decomposed number is the same value as the number in standard form (you do not multiply the decomposed values)	APPLY THE DISTRIBUTIVE PROPERTY and DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS
The student does not multiply correctly according to place value. For example, 300 × 60 ≠ 18 or 180 or 1800.	may not recognize the difference in multiplying the digits versus multiplying the values of the numbers (3 × 6 = 18; 300 is 100 times more than 3, and 60 is 10 times more than 6, therefore when you multiply them the product is 1,000 times more than 18; it is 18,000)	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION

3. Jules was helping her grandmother prep and plant part of her garden. Her grandmother wanted to plant green onions and carrots. Use the information in the image to help you answer the following questions.

   3.a. What is the total area of the carrot section in square feet?



CORRECT ANSWER

$$\begin{array}{r} 10 + 6 \\ \times \quad 7 \\ \hline 70 = 10 \times 7 \\ 42 = 6 \times 7 \\ \hline 112 = \text{Total Product} \end{array}$$

The total area of the carrot section is 112 square feet.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not decompose the factor correctly.	does not know how to decompose a number by place value	DECOMPOSE NUMBERS UP TO 1000
The student incorrectly multiplies the partial products.	does not correctly multiply two single-factors or one two-digit by single-digit factor	MULTIPLY A 2-DIGIT NUMBER BY A 1-DIGIT NUMBER WITHOUT REGROUPING
The student does not decompose factors, multiply partial products, then add partial products to determine a total product.	may not understand the procedure of decomposing the numbers, multiplying, and then adding the partial products	DECOMPOSE NUMBERS UP TO 1000 and DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS
The student is not clear on *what* the problem is asking.	does not recognize what the question is asking	SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES and USE REASONING FOR MULTIPLICATION AND DIVISION

3.b. A packet of carrot seeds contains 1200 seeds. If Jules plants 16 carrot seeds per square foot, will one packet of seeds be enough?

CORRECT ANSWER

```
      100 + 10 + 2
x       10 + 6
-------------------
 1000 = 100 × 10
  600 = 100 × 6
  100 = 10 × 10
   60 = 10 × 6
   20 = 2 × 10
+  12 = 2 × 6
-------------------
 1792 = Total Product
```

Jules' grandmother will need 1792 carrot seeds. No, one packet of seeds will not be enough.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not decompose the factor correctly.	does not know how to decompose a multi-digit number	DECOMPOSE NUMBERS UP TO 1000
The student incorrectly multiplies the partial products.	does not correctly multiply two single-factors or one two-digit by single-digit factor	MULTIPLY A 2-DIGIT NUMBER BY A 1-DIGIT NUMBER WITHOUT REGROUPING
The student does not break apart factors, multiply partial products, then add partial products to determine a total product.	may not understand the procedure of decomposing the numbers, multiplying, and then adding the partial products	DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS
The student multiplies 12,000 × 432, 12,000 × 2, or 432 × 2.	does not recognize what the question is asking	SOLVE PART/WHOLE PROBLEMS and USE REASONING FOR MULTIPLICATION AND DIVISION

3.c. The area of the section for the green onions is 105 square feet, and Jules' grandmother wants to plant 16 seeds in each square foot. A packet of green onion seeds contains 1000 seeds. How many seeds does Jules' grandmother want to plant in all? How many packets of green onion seeds will Jules' grandmother need to purchase?

CORRECT ANSWER

```
      100 + 5
x      10 + 6
-------------
1000 = 100 × 10
 600 = 100 × 6
  50 = 5 × 10
+ 30 = 5 × 6
-------------
1680 = Total Product
```

Jules' grandmother wants to plant 1680 seeds in all and will need to purchase two packets of green onion seeds.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not decompose the factors correctly.	does not understand how to decompose a multi-digit number by place value	DECOMPOSE NUMBERS UP TO 1000
The student does not multiply the factors correctly.	does not correctly multiply two single-factors or one two-digit by single-digit factor	MULTIPLY A 2-DIGIT NUMBER BY A 1-DIGIT NUMBER WITHOUT REGROUPING
The student does not add partial products to determine a total product.	may not understand that the partial products need to be added together for the total product, since the factors were decomposed	DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS
The student cannot identify what information (numbers) to use from the problem. Student may multiply the wrong numbers together or not answer the question. Student may just multiply any two numbers given.	does not recognize what the question is asking	SOLVE PART/WHOLE PROBLEMS and USE REASONING FOR MULTIPLICATION AND DIVISION

4. Describe the steps necessary to multiply a three-digit number times a two-digit number using partial products. Support your answer by creating and solving an example.

CORRECT ANSWER

1. Decompose the factors by place value.
2. Multiply the decomposed factors times each other. There should be six partial products.
3. Add the partial products together for the total product.

(Check student's example for accuracy.)

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student only provides an example, with no explanation of the steps required to solve the problem.	unable to explain the process used to solve the problem	EXPLAIN THE DISTRIBUTIVE PROPERTY and DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS
(In the student example) The student did not decompose the factors by place value	does not understand how to decompose a multi-digit number by place value	DECOMPOSE NUMBERS UP TO 1000
(In the student example) The student does not multiply the factors correctly.	does not correctly multiply two single-factors or one two-digit by single-digit factor	MULTIPLY A 2-DIGIT NUMBER BY A 1-DIGIT NUMBER WITHOUT REGROUPING
The student does not add partial products to determine a total product.	may not understand that the partial products need to be added together for the total product, since the factors were decomposed	DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS
The student is missing partial products or multiplies two values from the same decomposed number.	may not understand that a decomposed number is the same value as the number in standard form; does not multiply the decomposed values	APPLY THE DISTRIBUTIVE PROPERTY and DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS

5. Use the expression 576 × 54 to answer the following questions.

5.a. Decompose the factors in the expression.

CORRECT ANSWER

> 500, 70, 6 and 50, 4

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not correctly decompose the factors.	does not understand how to decompose a multi-digit number by place value	DECOMPOSE NUMBERS UP TO 1000

5.b. What are the partial products for the expression?

CORRECT ANSWER

500 × 50 = 25,000
500 × 4 = 2000
70 × 50 = 3500
70 × 4 = 280
6 × 50 = 300
6 × 4 = 24

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not correctly decompose the factors.	does not understand how to decompose a multi-digit number by place value	DECOMPOSE NUMBERS UP TO 1000
The student incorrectly multiplies the partial products.	does not correctly multiply two single-factors or one two-digit by single-digit factor	MULTIPLY A 2-DIGIT NUMBER BY A 1-DIGIT NUMBER WITHOUT REGROUPING and MULTIPLY A 3-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING
The student is missing partial products or multiplies two values from the same decomposed number.	may not understand that a decomposed number is the same value as the number in standard form; does not multiply the decomposed values	APPLY THE DISTRIBUTIVE PROPERTY and REPRESENT 3-DIGIT NUMERALS WITH EXPANDED FORM

5.c. What is the total product?

CORRECT ANSWER

25,000 + 2000 + 3500 + 280 + 300 + 24 = 31,104

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student did not add partial products to determine a total product.	may not understand that the partial products need to be added together for the total product, since the factors were decomposed	DETERMINE THE PRODUCT OF 2 FACTORS USING PARTIAL PRODUCTS

6. Answer the following questions using the standard algorithm.

6.a. Chris has his own food truck. On Mondays and Tuesdays, he parks his truck in the city near the museums and office buildings. Chris usually sells 302 tacos when he parks in the city. On Fridays and Saturdays, he parks his truck near the beach. Chris sells 9 times more tacos on the weekends than he does on Mondays and Tuesdays. How many tacos does Chris sell during the weekends?

CORRECT ANSWER

$$\begin{array}{r} 302 \\ \times \quad 9 \\ \hline 2718 \end{array}$$

Chris sells 2718 tacos during the weekends.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not regroup at all or regrouped incorrectly when multiplying.	unable to multiply with regrouping	MULTIPLY A 3-DIGIT NUMBER BY A 1-DIGIT NUMBER WITH REGROUPING and EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION
The student does not use the standard algorithm and only provides an alternate strategy.	does not know how to apply the standard algorithm to solve a multiplication problem	MULTIPLY A 3-DIGIT NUMBER BY A 1-DIGIT NUMBER WITH REGROUPING and USE REASONING FOR MULTIPLICATION AND DIVISION
The student does not answer or is not clear on *what* the problem is asking.	does not recognize what the question is asking	SOLVE MULTIPLICATIVE COMPARISON PROBLEMS and USE REASONING FOR MULTIPLICATION AND DIVISION

6.b. Marco wants to paint his game room green. Three of his walls are the same size; each wall is 295 inches long and 96 inches tall. How many square inches will Marco need to paint for all three walls?

CORRECT ANSWER

```
      295
    ×  96
     1770
  + 26550
    28320
  ×     3
   84960
```

One Wall = 28320

Three Walls = 84960

Marco will need to paint 84,960 square inches for all three walls.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not provide an alternate strategy.	may be unaware of how to solve using a different method	CALCULATE PRODUCTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM
The student does not regroup at all or regroups incorrectly when multiplying.	unable to multiply with regrouping	MULTIPLY A 3-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING and EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION
The student does not use the standard algorithm and only provides an alternate strategy.	does not know how to apply the standard algorithm to solve a multiplication problem	MULTIPLY A 3-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING and USE REASONING FOR MULTIPLICATION AND DIVISION
The student cannot identify what information (numbers) to use from the problem. Student may multiply the wrong numbers together or not answer the question. Student may just multiply any two numbers given.	does not recognize what the question is asking	SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES and USE REASONING FOR MULTIPLICATION AND DIVISION

6.c. Macey is creating a 10-slide digital presentation for school. She has 361 different fonts available to use and 376 different color options. How many possible font and color combinations does Macey have to choose from when making her presentation?

CORRECT ANSWER

$$\begin{array}{r} 361 \\ \times\ 376 \\ \hline 2166 \\ 25270 \\ +\ 108300 \\ \hline 135736 \end{array}$$

Macey can choose from 135,736 different font and color combinations.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not provide an alternate strategy.	may be unaware of how to solve using a different method	CALCULATE PRODUCTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM
The student does not regroup at all or correctly when multiplying.	unable to multiply with regrouping	MULTIPLY A 3-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING and EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND MULTIPLICATION
The student does not use the standard algorithm and only provides an alternate strategy.	does not know how to apply the standard algorithm to solve a multiplication problem	MULTIPLY A 3-DIGIT NUMBER BY A 2-DIGIT NUMBER WITH REGROUPING and USE REASONING FOR MULTIPLICATION AND DIVISION
The student cannot identify what information (numbers) to use from the problem. Student may multiply the wrong numbers together or not answer the question. Student may just multiply any two numbers given.	does not recognize what the question is asking	SOLVE CARTESIAN PRODUCTS and USE REASONING FOR MULTIPLICATION AND DIVISION